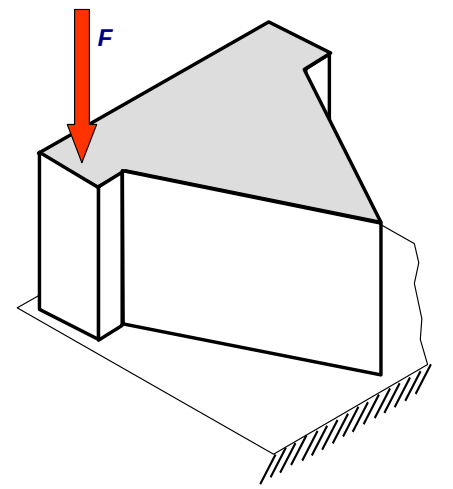
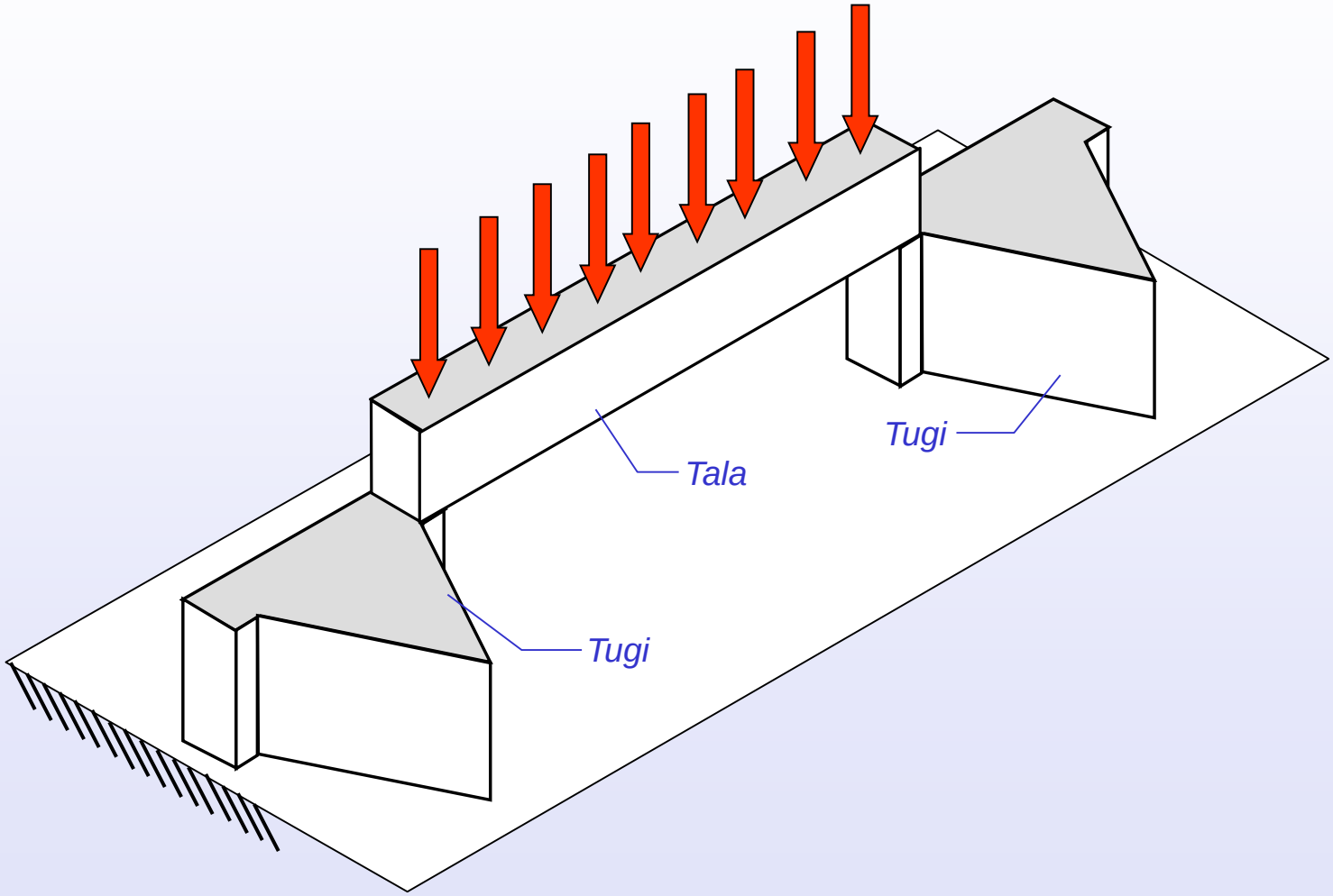
TUGEVUSÕPETUS

Ekstsentriline surve



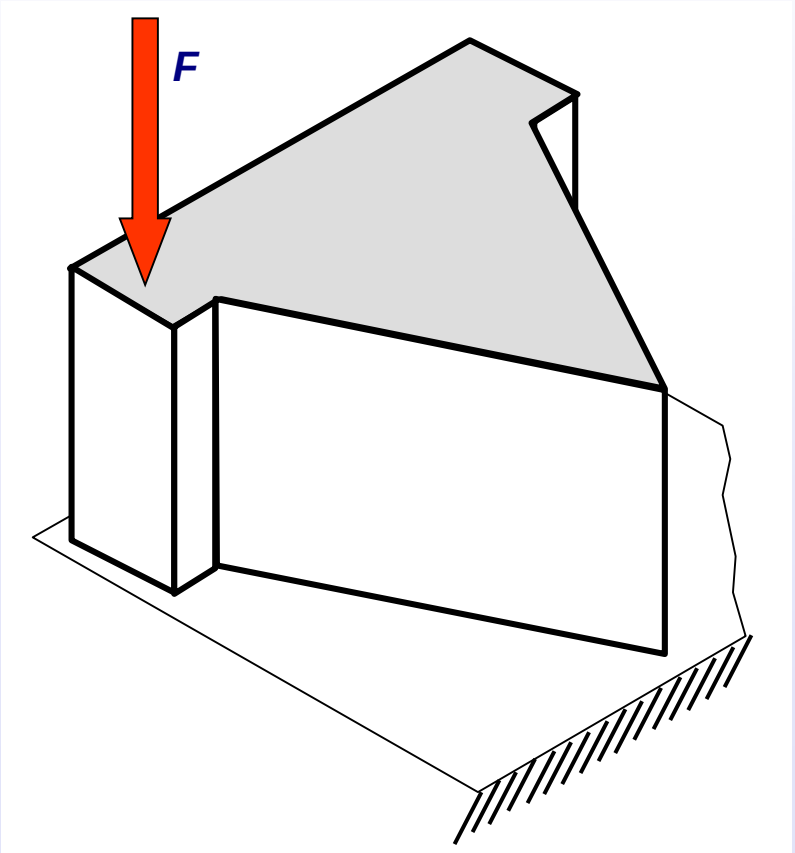
1. Ülesande püstitus ja algandmed

Tugedega konstruktsioon

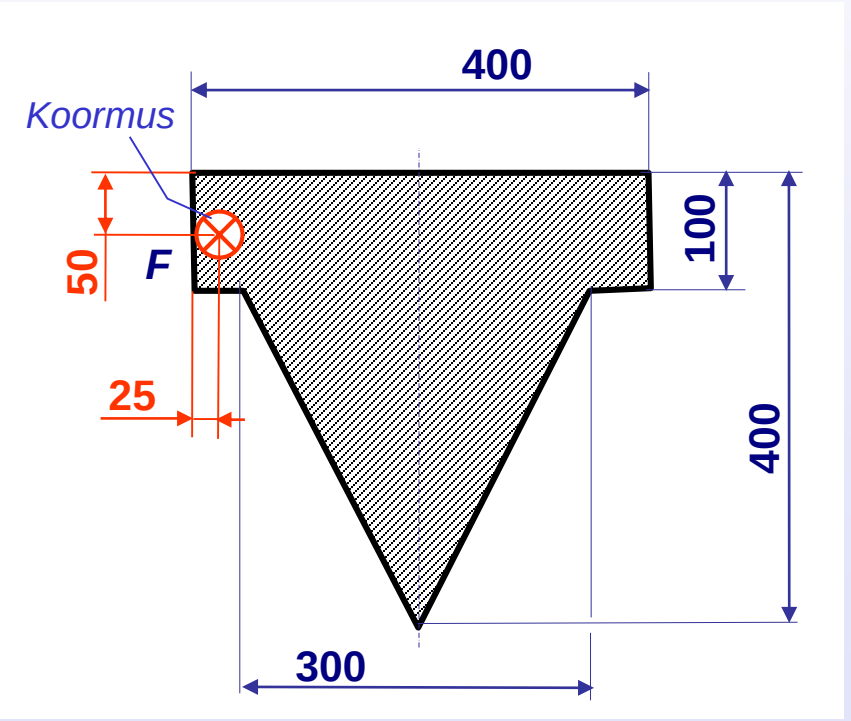


Lühike ühtlane surutud tugi

Arvutada lühikesele toele koormuse *F* suurim lubatav väärtus !



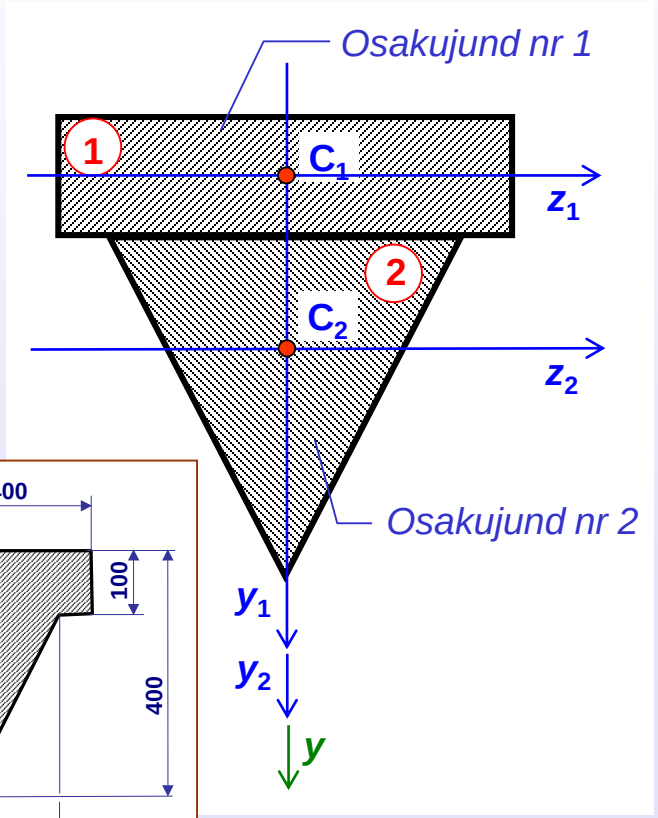
Toe ristlõike kuju ja koormuse asend

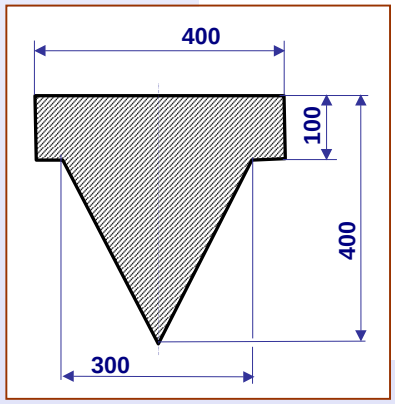


2. Toe kesk-peatasandid

2.1. Ristlõike osakujundid ja teljestikud

Liitkujund koosneb 2-st osakujundist





Osakujundid

Osakujund nr 1 = ristkülik pinnakeskmega C_1

Osakujund nr 2 = võrdhaarne kolmnurk pinnakeskmega C_2

Teljestikud

y_1z_1 = osakujundi nr 1 kesk-peateljestik

y_2z_2 = osakujundi nr 2 kesk-peateljestik

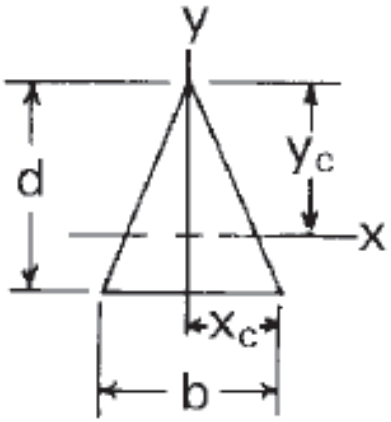
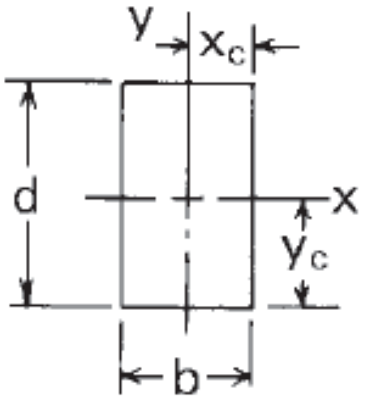
Telg $y_1 \equiv y_2$ = osakujundi nr 1 sümmeetriatelg
= osakujundi nr 2 sümmeetriatelg
= liitkujundi SÜMMEETRIATELG

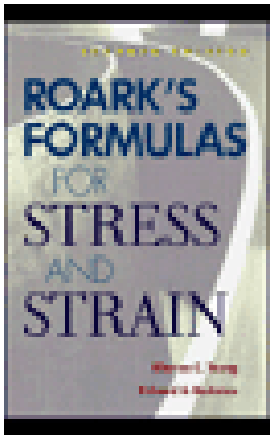
Telg $y_1 \equiv y_2 \equiv y$ = liitkujundi KESKPEATELG

Liitkujundi teine keskpeatelg (z) on esimesega (y) risti ja läbib pinnakeset

2.2. Osakujundite andmed

Lihtkujundite andmed on toodud käsiraamatutes

Isosceles triangle 	$A = \frac{bd}{2}$ $y_c = \frac{2}{3}d$ $x_c = \frac{b}{2}$	$I_x = \frac{1}{36}bd^3$ $I_y = \frac{1}{48}db^3$ $I_x > I_y$ if $d > 0.866b$ $r_x = 0.2357d$ $r_y = 0.2041b$
Rectangle 	$A = bd$ $y_c = \frac{d}{2}$ $x_c = \frac{b}{2}$	$I_x = \frac{1}{12}bd^3$ $I_y = \frac{1}{12}db^3$ $I_x > I_y$ if $d > b$ $r_x = 0.2887d$ $r_y = 0.2887b$



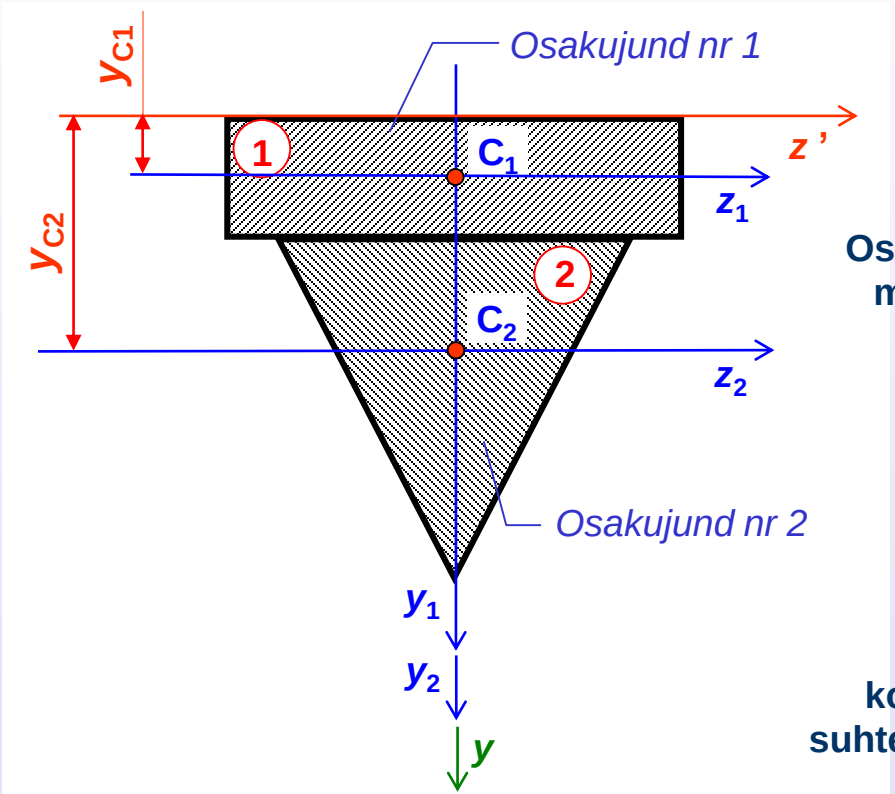
2.3. Ristlõike pinnakeskme asukoht (1)

Liitkujundi pinnakeskme koordinaat telje *z* ' suhtes:

$$y_C = \frac{S_{z'}}{A}$$

Liitkujundi staatiline moment telje *z*' suhtes

Liitkujundi pindala



Liitkujundi staatiline moment telje *z* ' suhtes

$$S_{z'} = S_{z'}^{(1)} + S_{z'}^{(2)}$$

Osakujundi nr 1 staatiline moment telje *z*' suhtes

Osakujundi nr 2 staatiline moment telje *z*' suhtes

Osakujundite staatilised momendid

$$S_z^{(1)} = y_{C1} A^{(1)}$$

$$S_{z'}^{(2)} = y_{C2} A^{(2)}$$

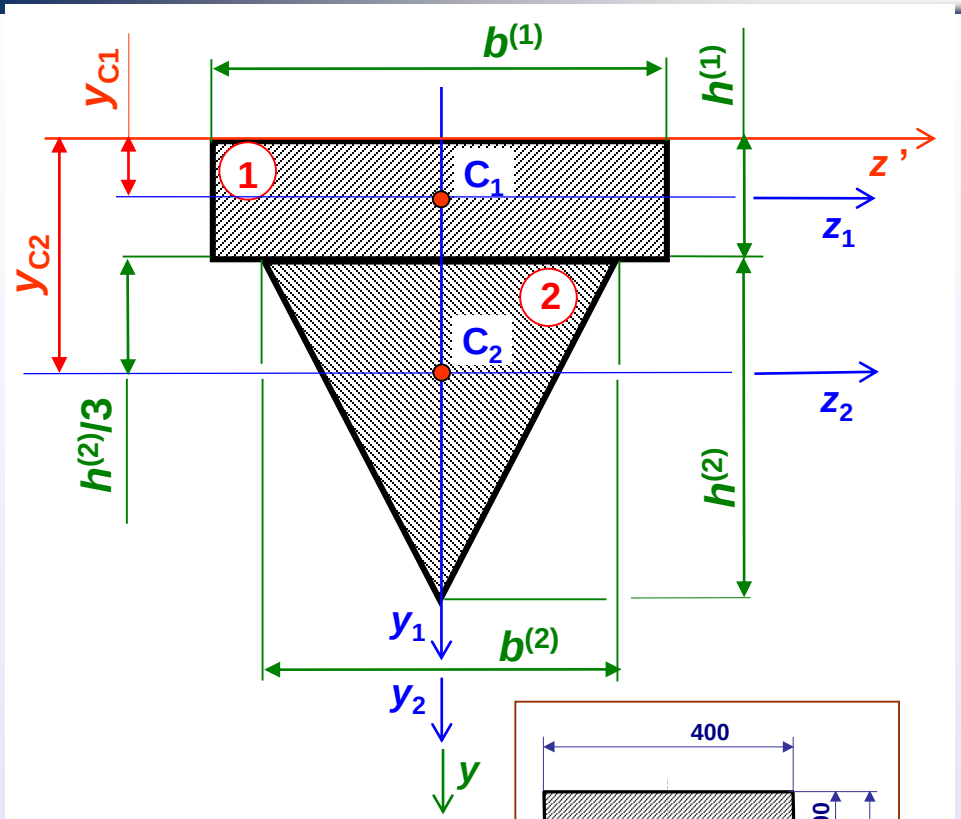
Osakujundi nr 1 pinnakeskme C_1 koordinaat telje *z* ' suhtes (*y*-koordinaat)

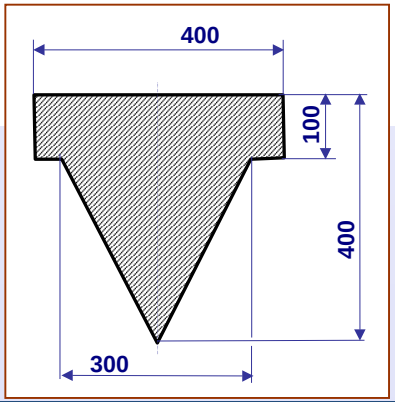
Osakujundi nr 1 pindala

Osakujundi nr 2 pinnakeskme C_2 koordinaat telje *z* ' suhtes (*y*-koordinaat)

Osakujundi nr 2 pindala

2.3. Ristlõike pinnakeskme asukoht (2)





Osakujundite pinnakeskmete koordinaadid

$$y_{C1} = \frac{h^{(1)}}{2} = \frac{100}{2} = 50 \text{ mm}$$

$$y_{C2} = h^{(1)} + \frac{h^{(2)}}{3} = 100 + \frac{400 - 100}{3} = 200 \text{ mm}$$

NB! Koordinaadid on MÄRGIGA suurused

Osakujundite pindalad

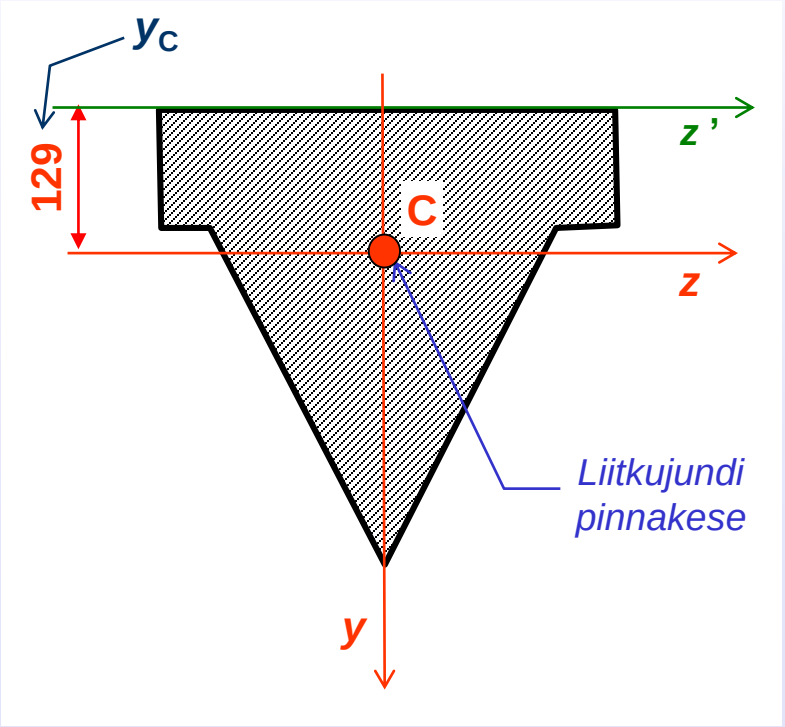
$$A^{(1)} = b^{(1)} h^{(1)} = 400 \cdot 100 = 40000 \text{ mm}^2$$

$$A^{(2)} = \frac{b^{(2)} h^{(2)}}{2} = \frac{300 \cdot (400 - 100)}{2} = 45000 \text{ mm}^2$$

2.3. Ristlõike pinnakeskme asukoht (3)

Liitkujundi pinnakeskme *z* - koordinaat

$$y_C = \frac{S_{z'}}{A} = \frac{y_{C1}A^{(1)} + y_{C2}A^{(2)}}{A^{(1)} + A^{(2)}} = \frac{50 \cdot 40000 + 200 \cdot 45000}{40000 + 45000} = 129{,}4 \approx 129 \text{ mm}$$

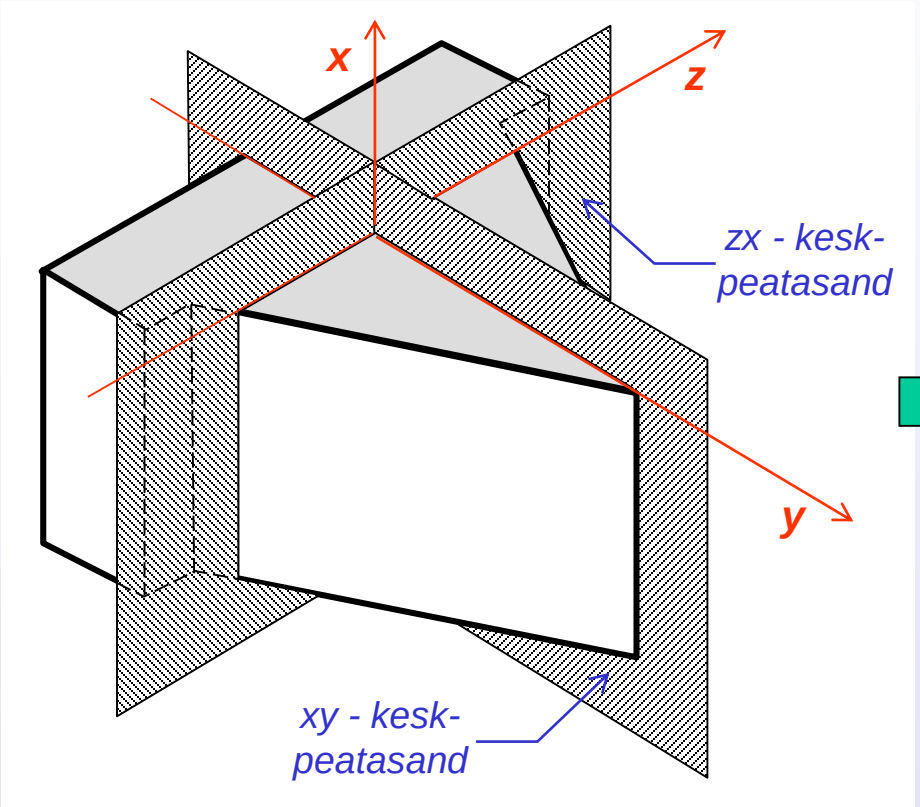


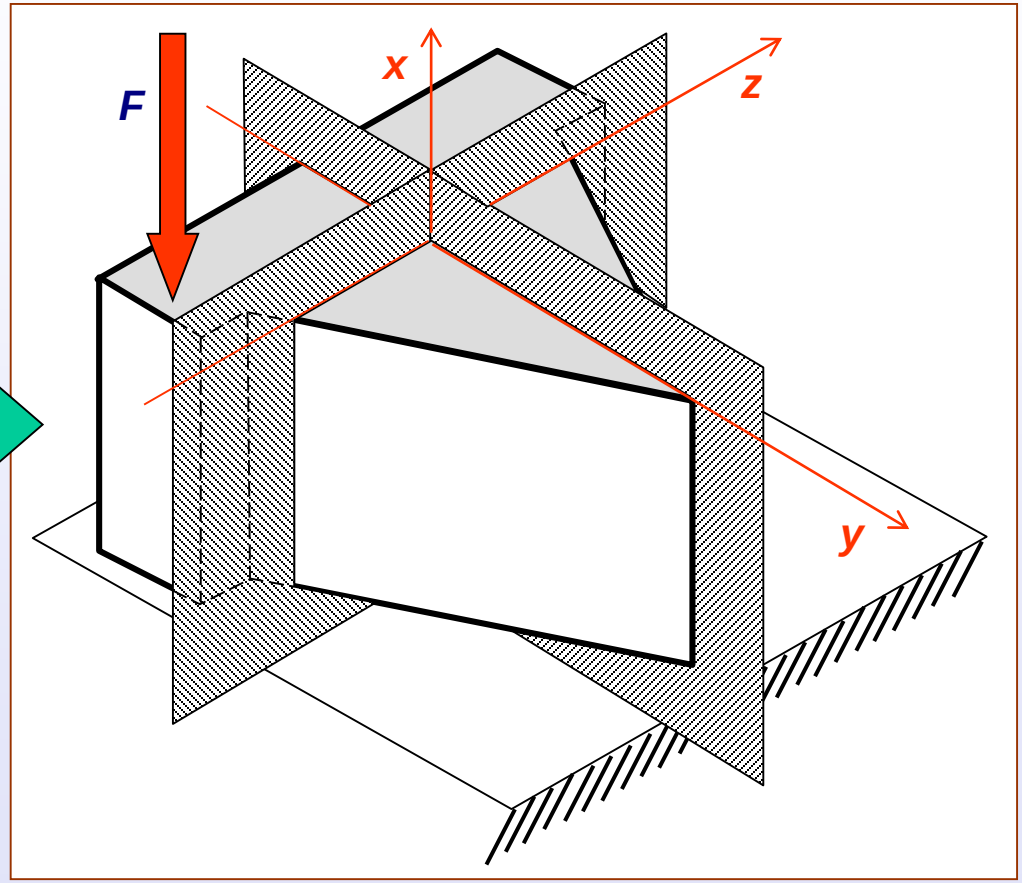
NB! Koordinaat on MÄRGIGA suurus

Teljestik *yz* = liitkujundi KESK-PEATELJESTIK

Ristlõike pinnakese C on punkt, mida läbib varda telg

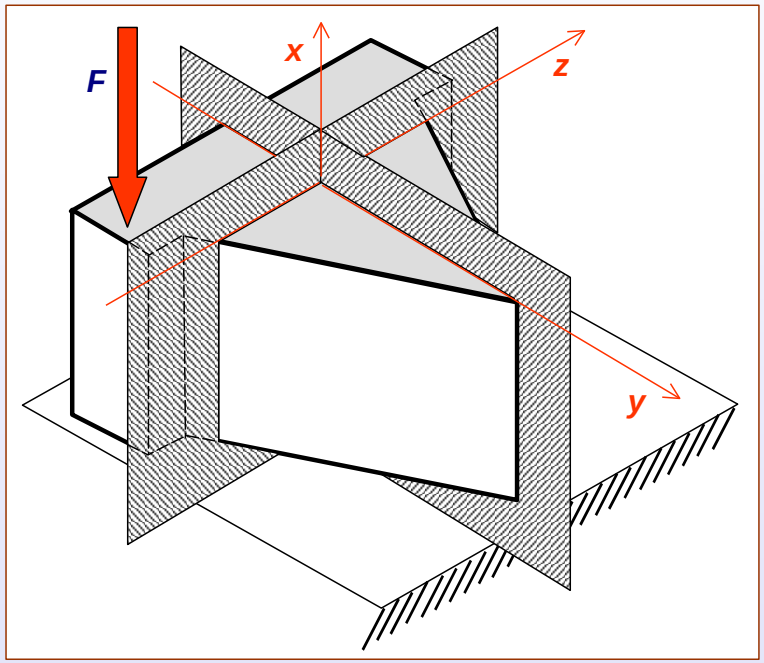
2.4. Toe kesk-peatasandid



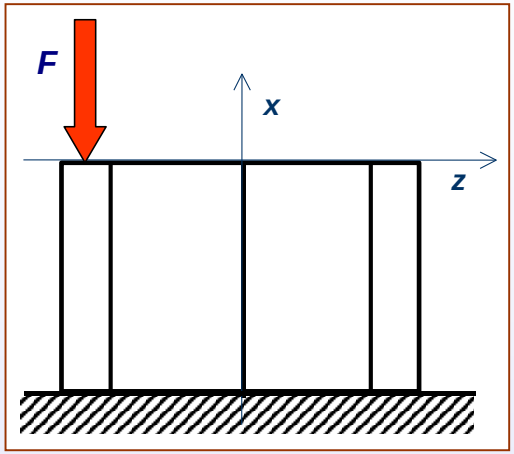


3. Toe sisejõudude analüüs

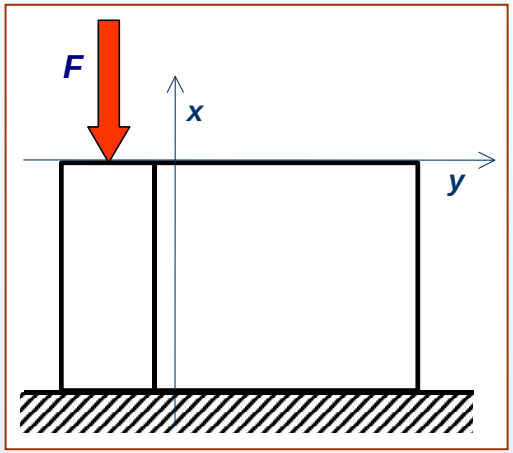
3.1. Koormatud toe kolmvaade



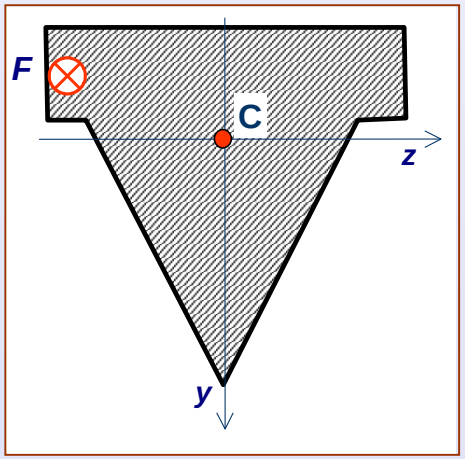
Vaade eest



Vaade vasakult

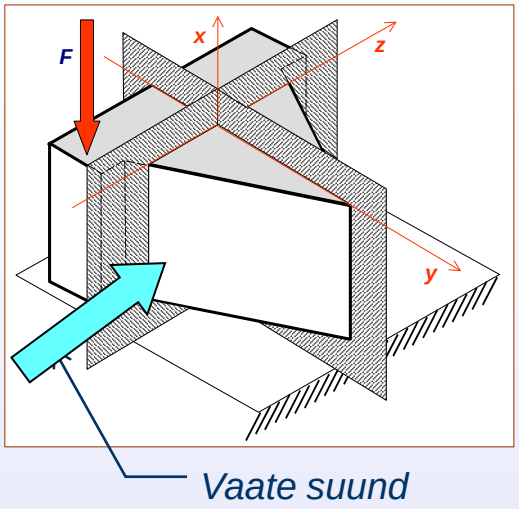


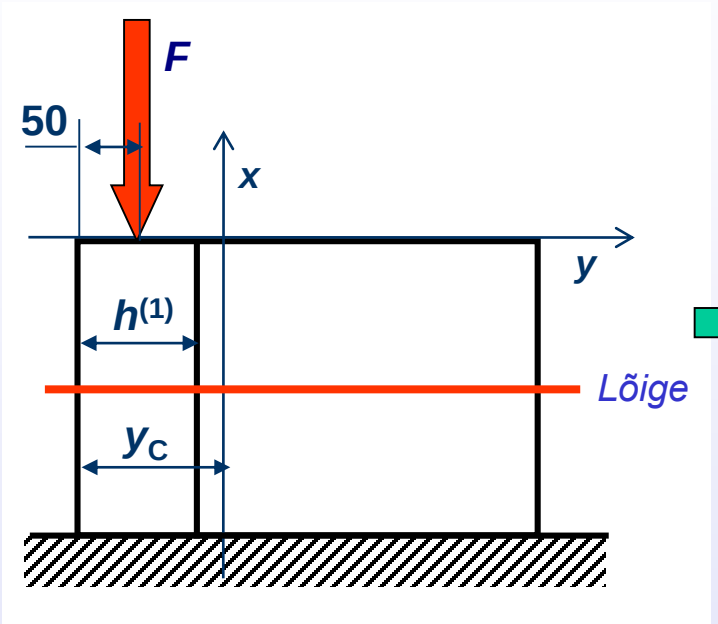
Vaade ülalt



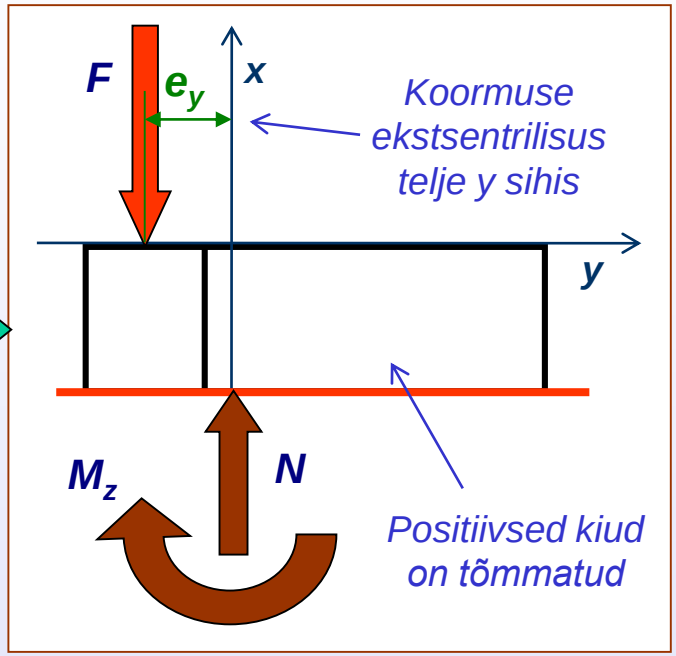
3.2. Sisejõud kesk-peatasandis *xy*

Sisejõud arvutatakse LÕIKEMEETODIGA





Lõike sisejõud



Koormuse ekstsentrilisus

$$e_y = y_C - 50 = 129 - 50 = 79 \text{ mm} = \underline{0{,}079 \text{ m}}$$

Tasakaaluvõrrandid

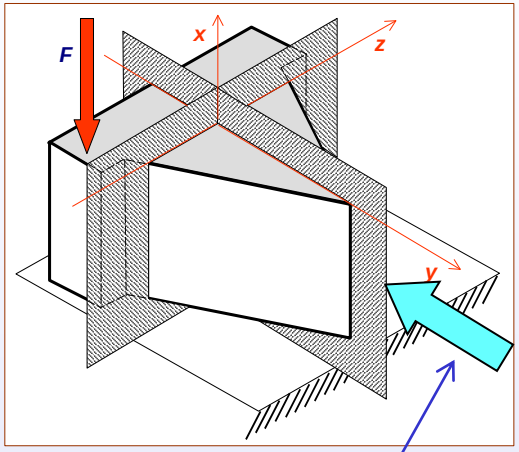
$$\sum F = 0 \Rightarrow F - N = 0 \Rightarrow \underline{N = F \ (-)}$$

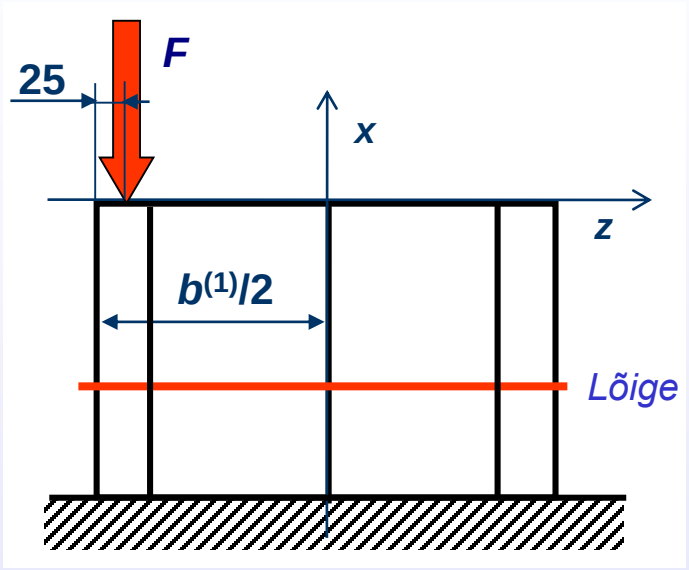
$$\sum \mathcal{M}_z = 0 \Rightarrow Fe_y - M_z = 0 \Rightarrow M_z = Fe_y = F \cdot 0{,}079 \Rightarrow \underline{M_z = 0{,}079F \ (+)}$$

Sisejõud ei sõltu koordinaadist *x*

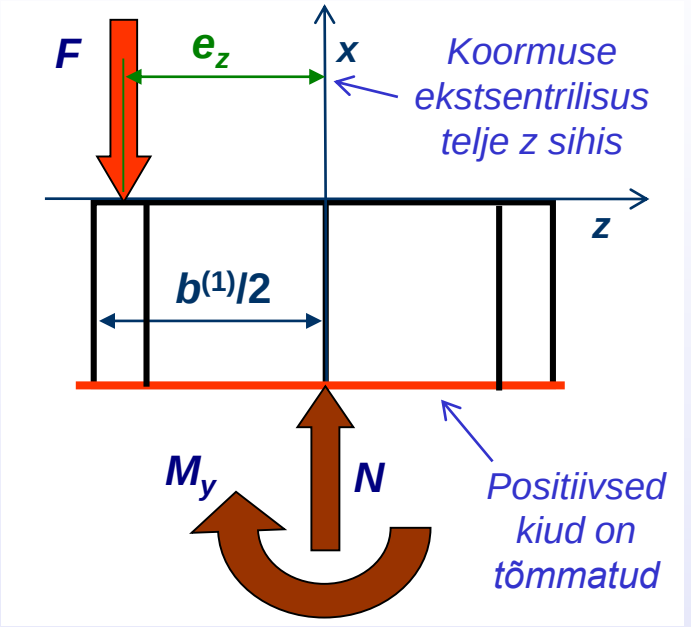
3.3. Sisejõud kesk-peatasandis *zx*

Sisejõud arvutatakse LÕIKEMEETODIGA





Lõike sisejõud



NB! *See on sama sisejõud N, mis tasandis xy*

Tasakaaluvõrrandid

$$\sum F = 0 \Rightarrow F - N = 0 \Rightarrow \underline{N = F\ (-)}$$

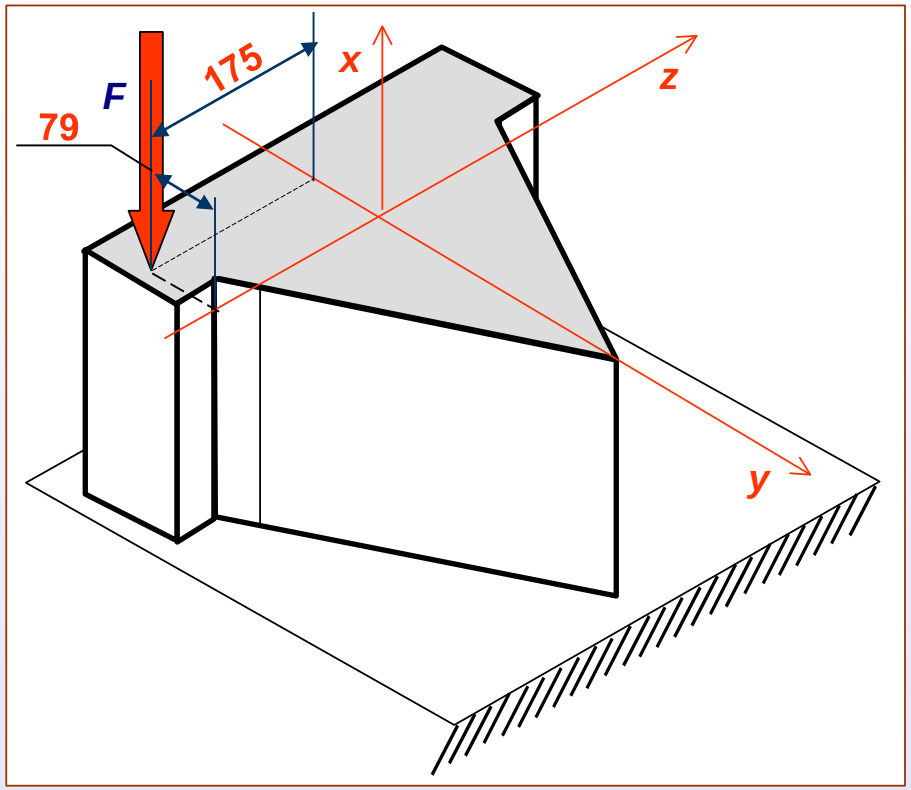
Koormuse ekstsentrilisus

$$e_z = \frac{b^{(1)}}{2} - 25 = \frac{400}{2} - 25 = 175\ \text{mm} = \underline{0{,}175\ \text{m}}$$

$$\sum \mathcal{M}_y = 0 \Rightarrow Fe_z - M_y = 0 \Rightarrow M_y = Fe_z = F \cdot 0{,}175 \Rightarrow \underline{M_y = 0{,}175F\ (+)}$$

Sisejõud ei sõltu koordinaadist *x*

3.4. Toe tööseisund



Ekstsentriliselt surutud tugi on ÜHTLASELT koormatud
(kõikides ristlõigetes on sisejõudude väärtused samad)

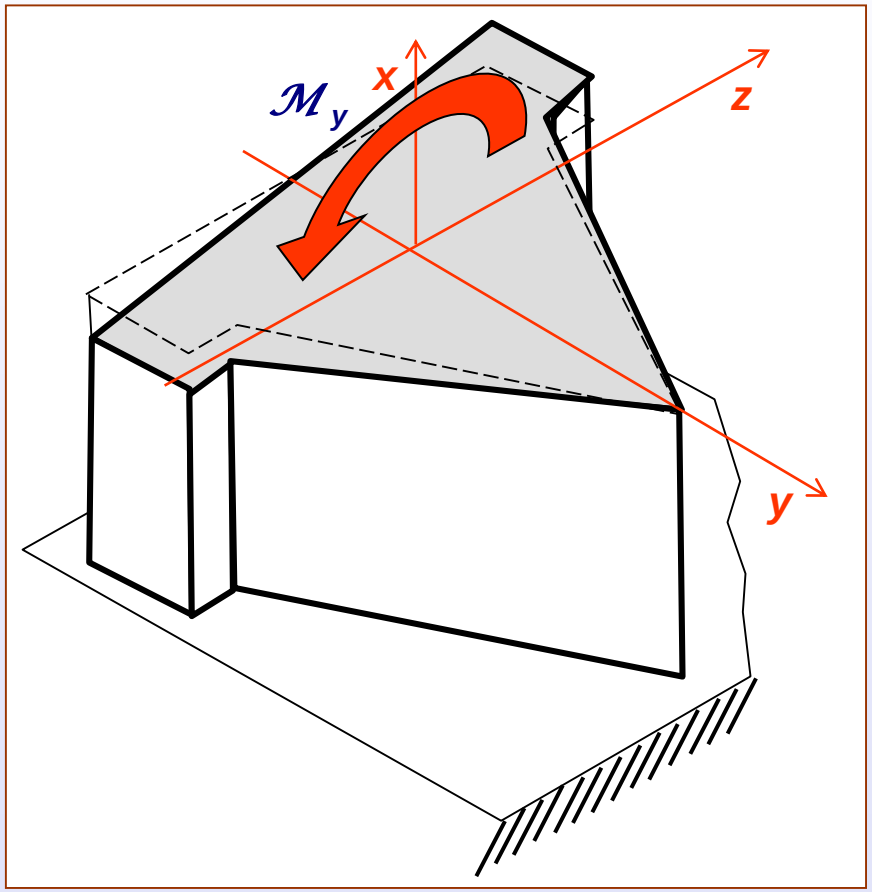
Ekstsentrilise surve sisejõud ristlõigetes

$$\begin{cases} N = F\ (-) \\ M_y = 0{,}175F\ (+) \\ M_z = 0{,}079F\ (+) \end{cases}$$

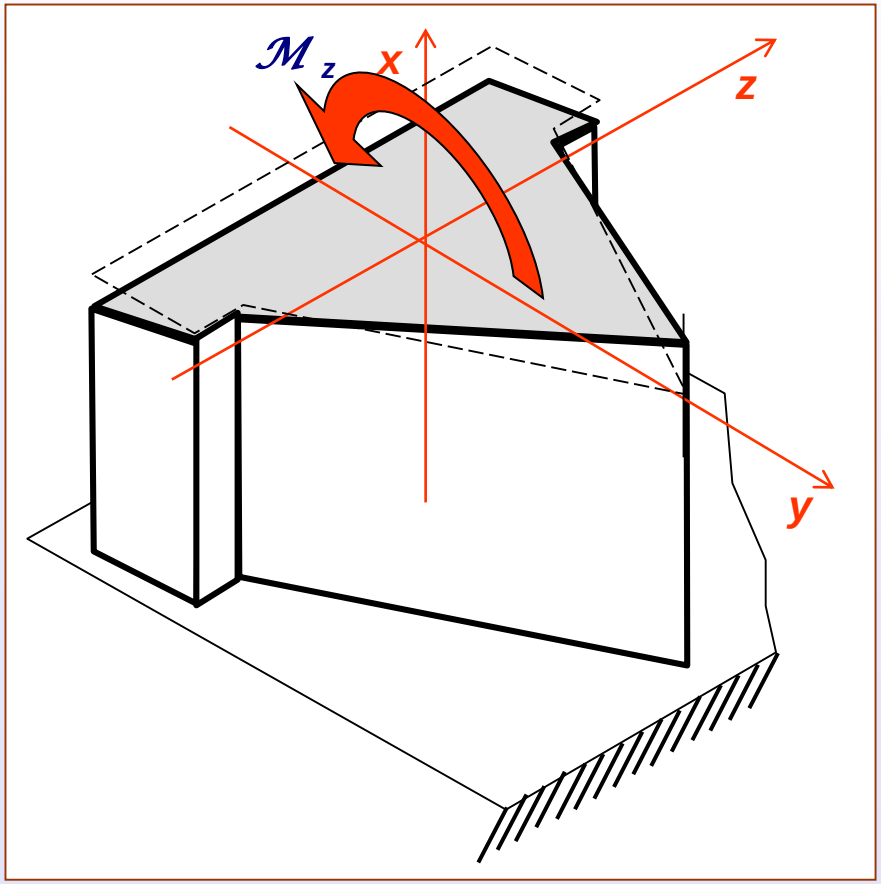
Ekstsentriline pike = PIKKE ja PAINDE koosmõju

3.5. Toe deformatsioonid (1)

Paine ümber telje *y*

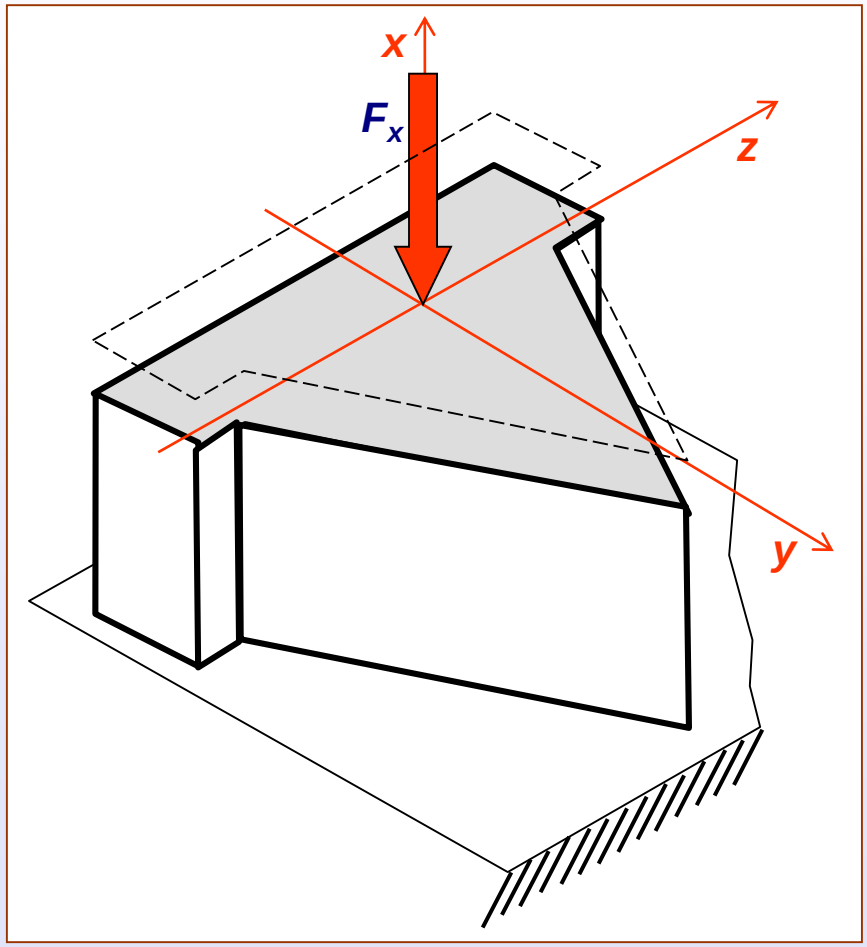


Paine ümber telje *z*

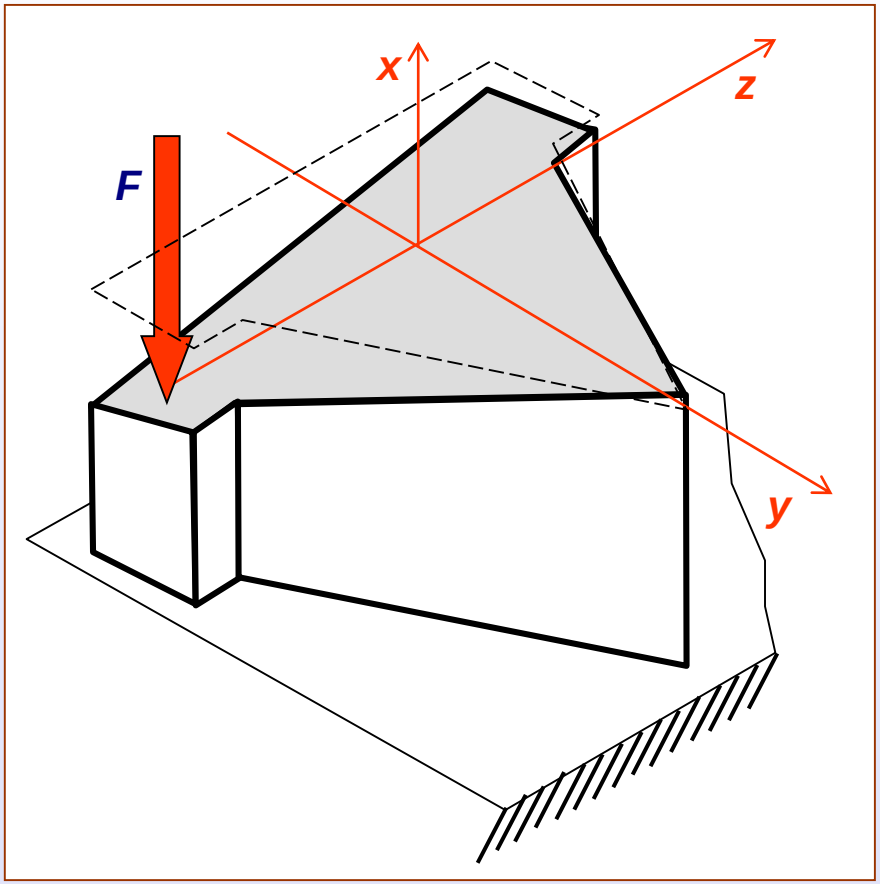


3.5. Toe deformatsioonid (2)

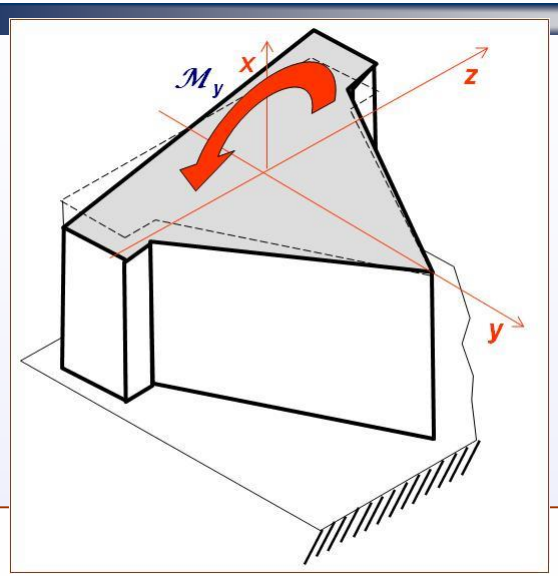
Surve



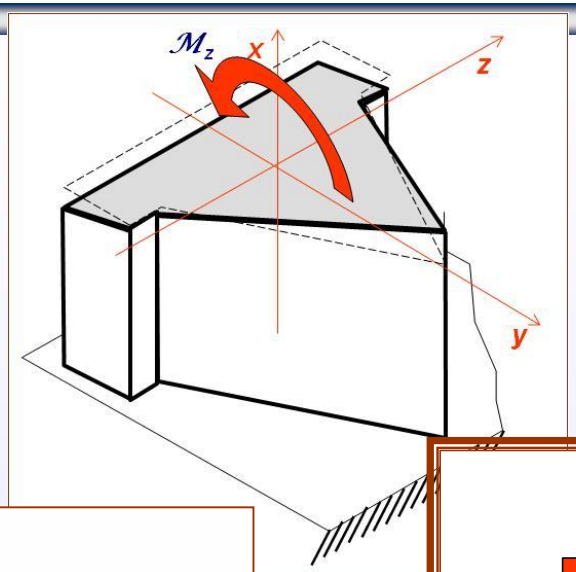
Kolme deformatsiooni summa



3.5. Toe deformatsioonid (3)



+



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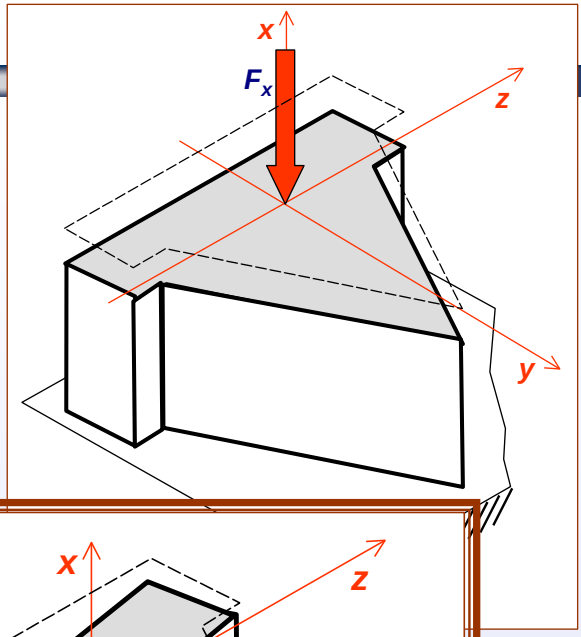
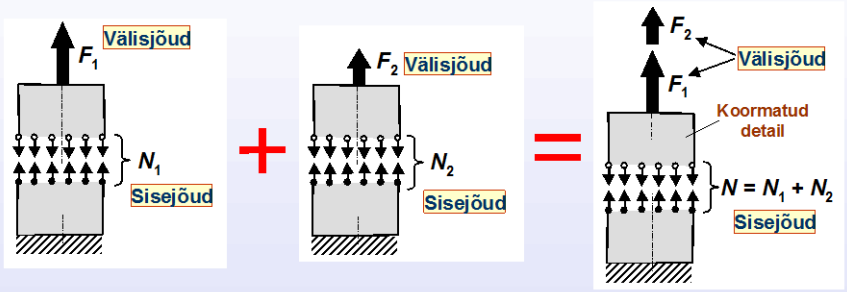
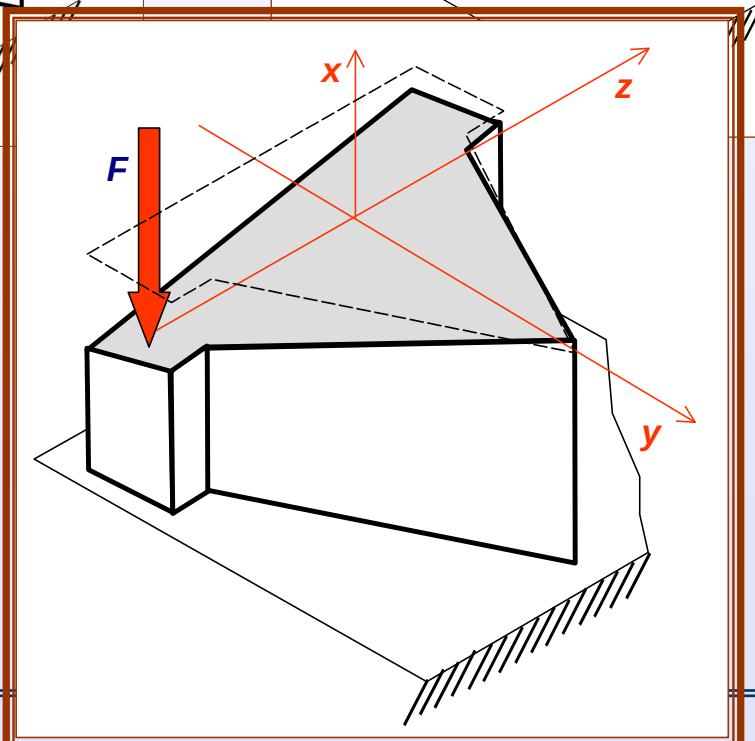


Jõu mõju sõltumatuse printsiip

Koormuste süsteemi mõju detailile = üksikute koormuste mõjude summa



Detailile <u>LISATUD</u> koormustest tulenevad sisejõudude (ja deformatsioonide) muutused
<u>EI SÕLTU</u> detailile varem rakendatud koormusest



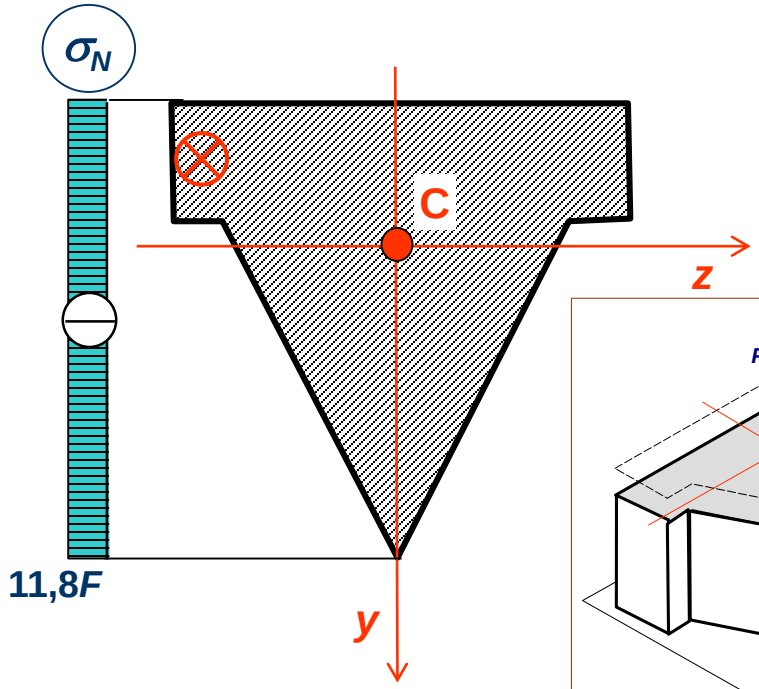
4. Toe ristlõike pingete laotused

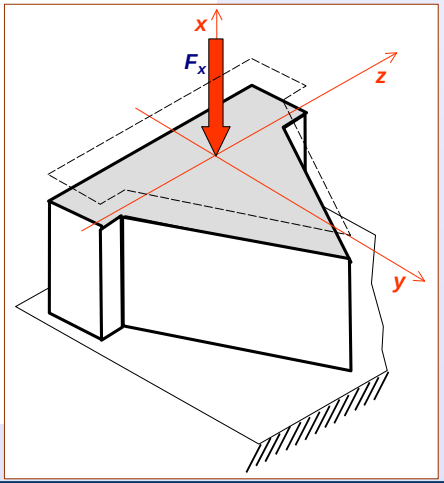
4.1. Pikkepinge toe ristlõigetes

Ristlõike pikkepinge laotus

$$\sigma_N = \frac{N}{A} = \frac{-F}{A^{(1)} + A^{(2)}} = \frac{-F}{40000 \cdot 10^{-6} + 45000 \cdot 10^{-6}} = -11{,}76F \approx -11{,}8F$$

Pikkepinge funktsioon toe ristlõikes

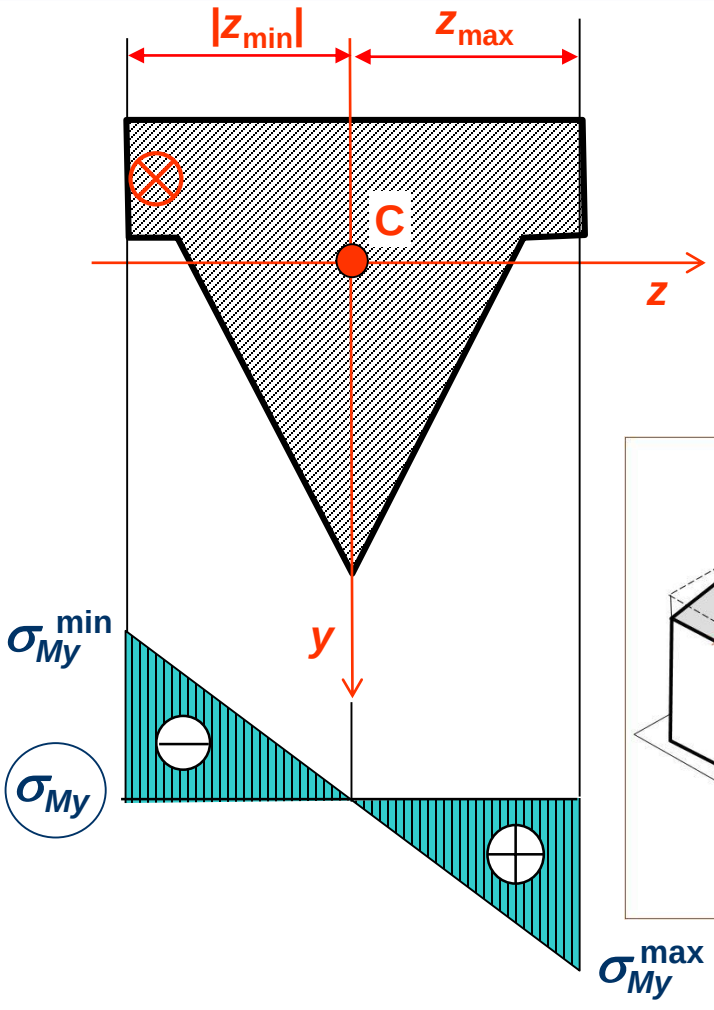


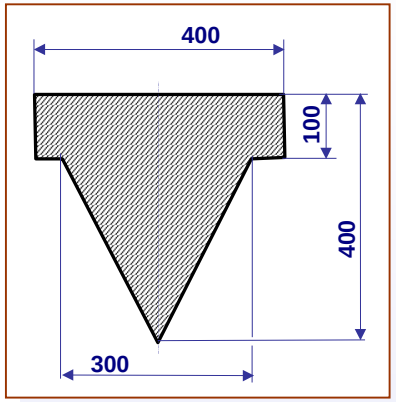


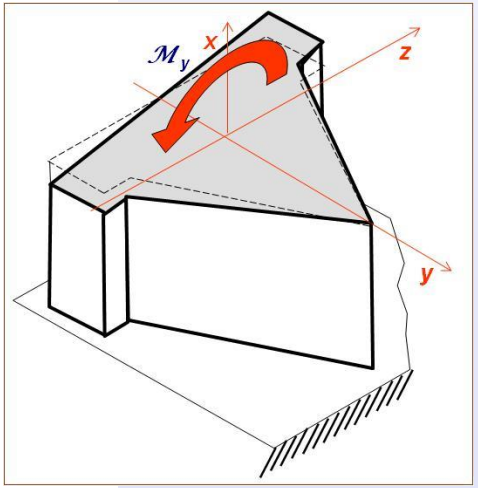
Pikkepinge laotus sisepinnal on ühtlane

Kõikides sisepinna punktides on pikkepinge väärtus sama

4.2. Paindepinge momendist M_y toe ristlõigetes







Ristlõike paindepinge laotus

$$\sigma_{My} = \frac{M_y}{I_y} z$$

- M_y — Ristlõike paindemoment
- z — Punkti koordinaat
- I_y — Ristlõike telg-inertsimoment

Ristlõike suurimad paindepinged

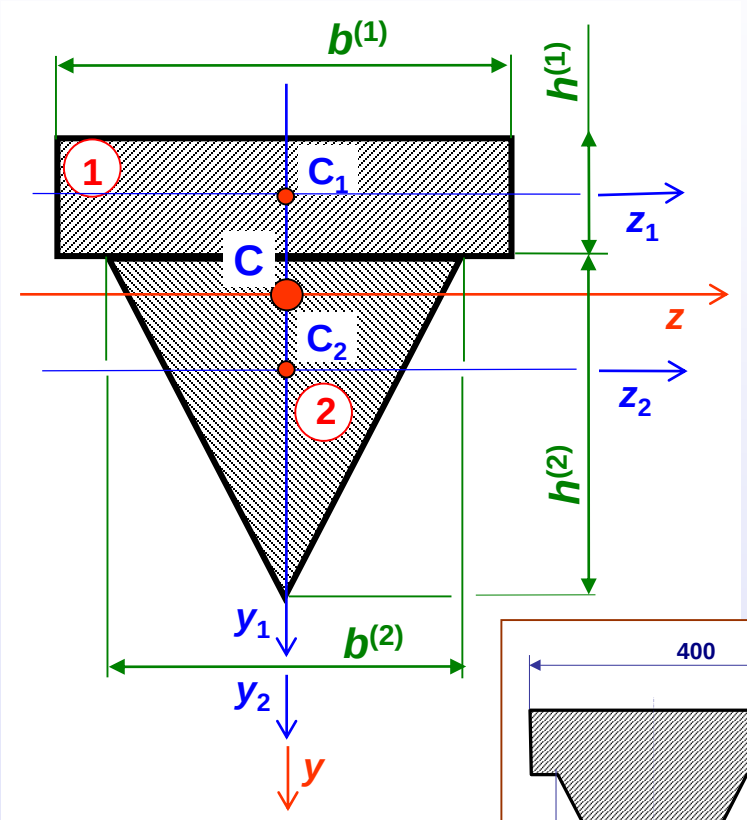
$$\sigma_{My}^{max} = \left|\sigma_{My}^{min}\right| = \frac{M_y}{I_y} z_{max} = \frac{M_y}{I_y} \left|z_{min}\right|$$

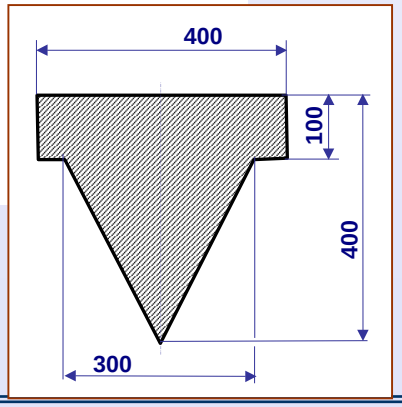
z_{max} = 200 mm = 0,2 m

z_{min} = - 200 mm = - 0,2 m

4.2.1. Ristlõike inertsimoment telje *y* suhtes

Ristlõike osakujundid





Inertsimoment telje *y* suhtes

$$I_y = I_y^{(1)} + I_y^{(2)}$$

Osakujundi nr. 2 inertsimoment telje *y* suhtes

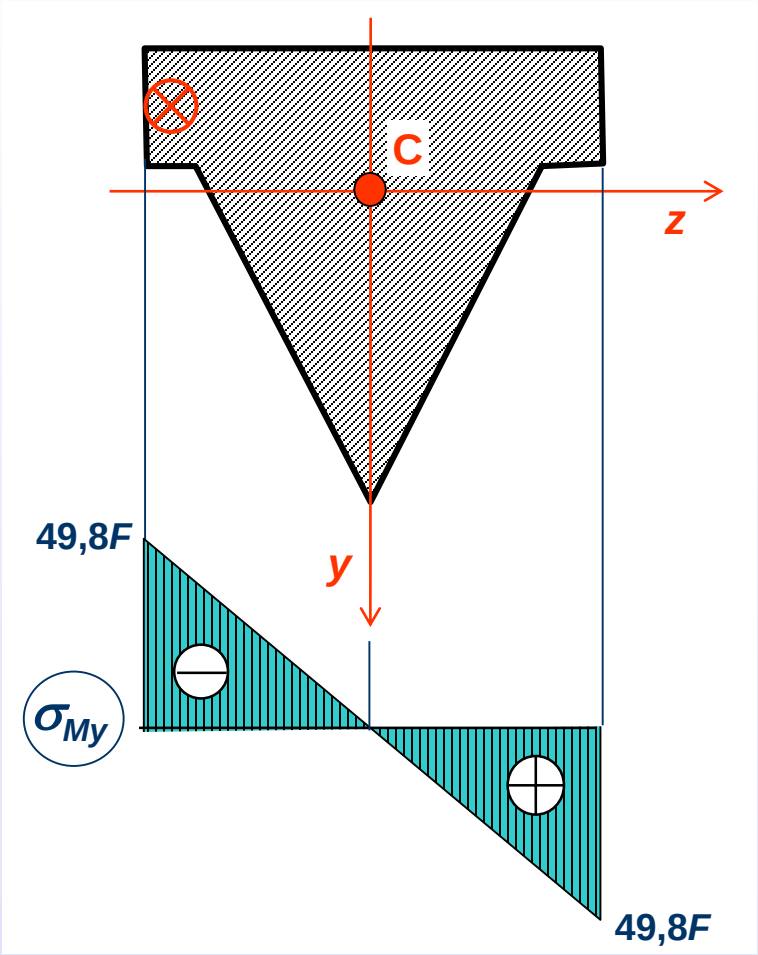
Osakujundi nr. 2 inertsimoment telje *y* suhtes

$$I_y^{(1)} = \frac{b^{(1)3} h^{(1)}}{12} = \frac{400^3 \cdot 100}{12} = 533{,}3 \cdot 10^6 \text{ mm}^4 \approx 5{,}33 \cdot 10^{-4} \text{ m}^4$$

$$I_y^{(2)} = \frac{b^{(2)3} h^{(2)}}{48} = \frac{300^3 \cdot 300}{48} = 168{,}7 \cdot 10^6 \text{ mm}^4 \approx 1{,}69 \cdot 10^{-4} \text{ m}^4$$

$$I_y = I_y^{(1)} + I_y^{(2)} = 5{,}33 \cdot 10^{-4} + 1{,}69 \cdot 10^{-4} = 7{,}02 \cdot 10^{-4} \text{ m}^4$$

4.2.2. Ristlõike paindepinge σ_{My} funktsioon



Ristlõike paindepinge σ_{My} laotus

$$\sigma_{My} = \frac{M_y}{I_y} z = \frac{0{,}175F}{7{,}02 \cdot 10^{-4}} z = 249{,}2Fz \approx 249Fz$$

Paindepinge funktsioon toe ristlõikes

Ristlõike suurimad paindepinged

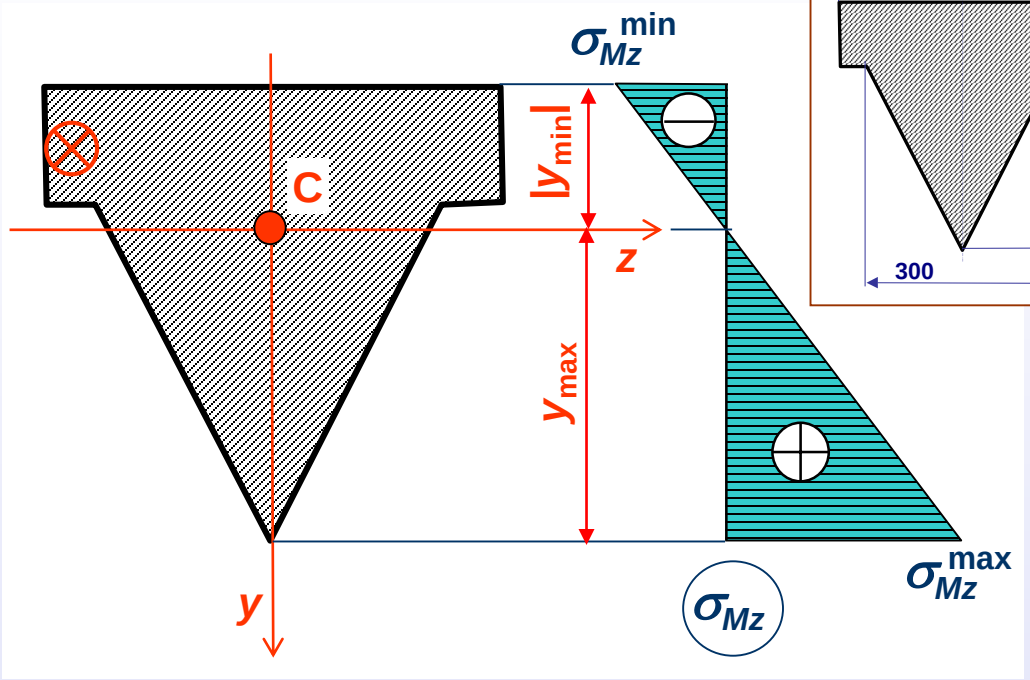
$$\sigma_{My}^{\max} = 249Fz_{\max} = 249 \cdot F \cdot 0{,}2 = 49{,}8F$$

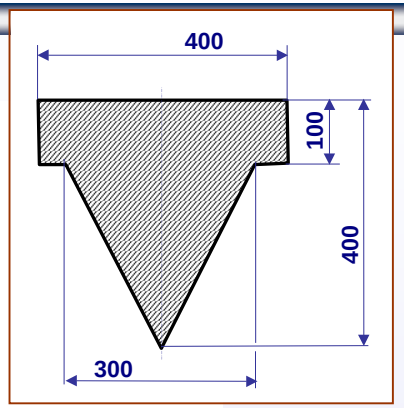
See on tõmbepinge (+)

$$\sigma_{My}^{\min} = 249Fz_{\min} = 249 \cdot F \cdot (-0{,}2) = -49{,}8F$$

See on survepinge (-)

4.3. Paindepinge momendist M_z toe ristlõigetes





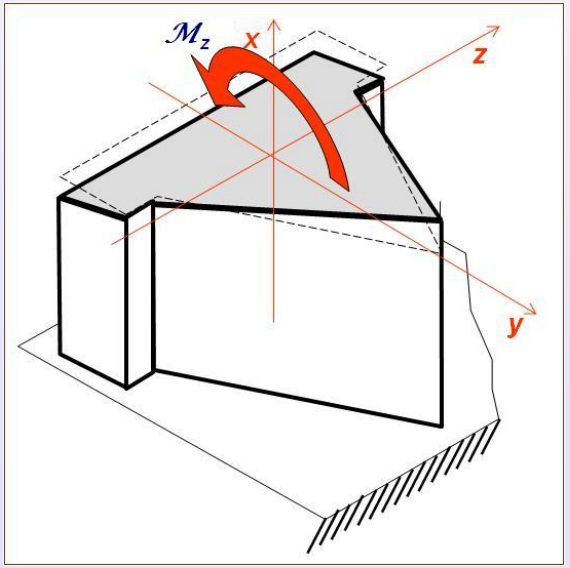
Ristlõike paindepinge laotus

$$\sigma_{Mz} = \frac{M_z}{I_z} y$$

Ristlõike telg-inertsimoment

Punkti koordinaat

Ristlõike paindemoment



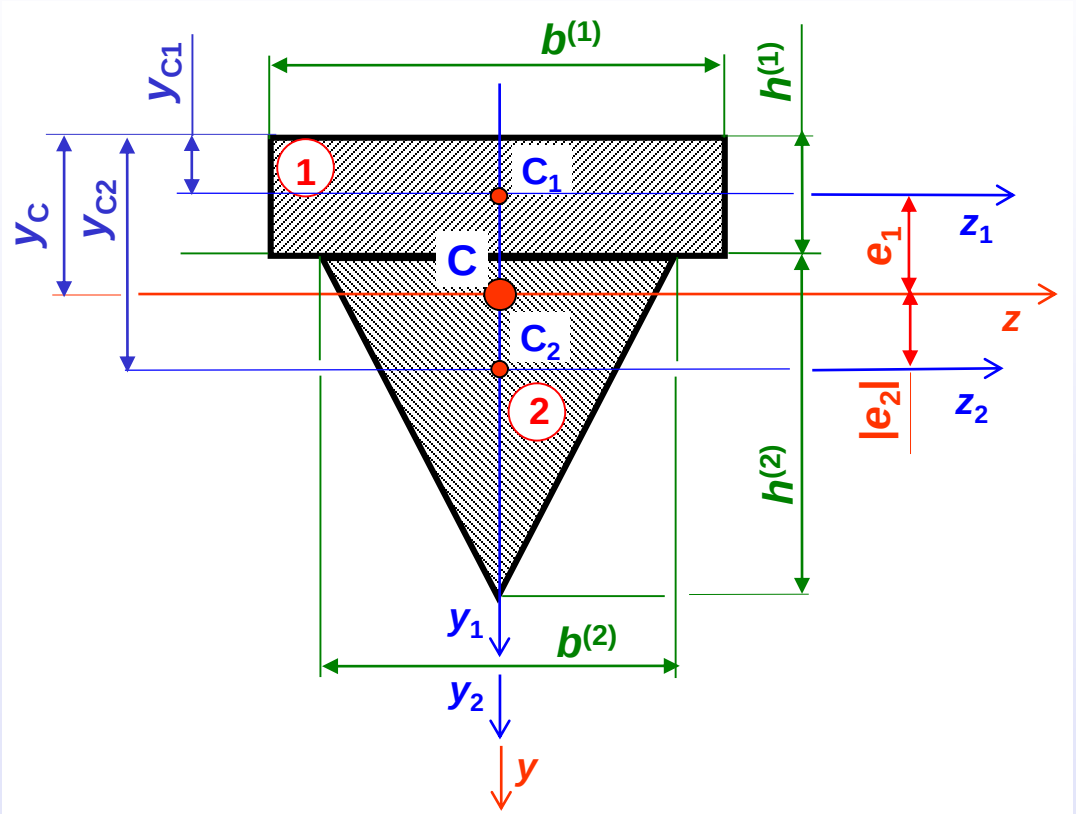
Ristlõike suurimad paindepinged

$$\sigma_{Mz}^{\max} = \frac{M_z}{I_z} y_{\max}$$

$$\sigma_{Mz}^{\min} = \frac{M_z}{I_z} y_{\min}$$

4.3.1. Ristlõike inertsimoment telje *z* suhtes (1)

Ristlõike osakujundid



Inertsimoment telje *z* suhtes

$$I_z = I_z^{(1)} + I_z^{(2)}$$

Osakujundi nr. 2 inertsimoment telje *z* suhtes

Osakujundi nr. 2 inertsimoment telje *z* suhtes

Rööpsete telgede teoreem osakujundile

$$I_M = I_K + e^2 A$$

Mitte-kesktelg *z*

Kesktelg z_1 või z_2

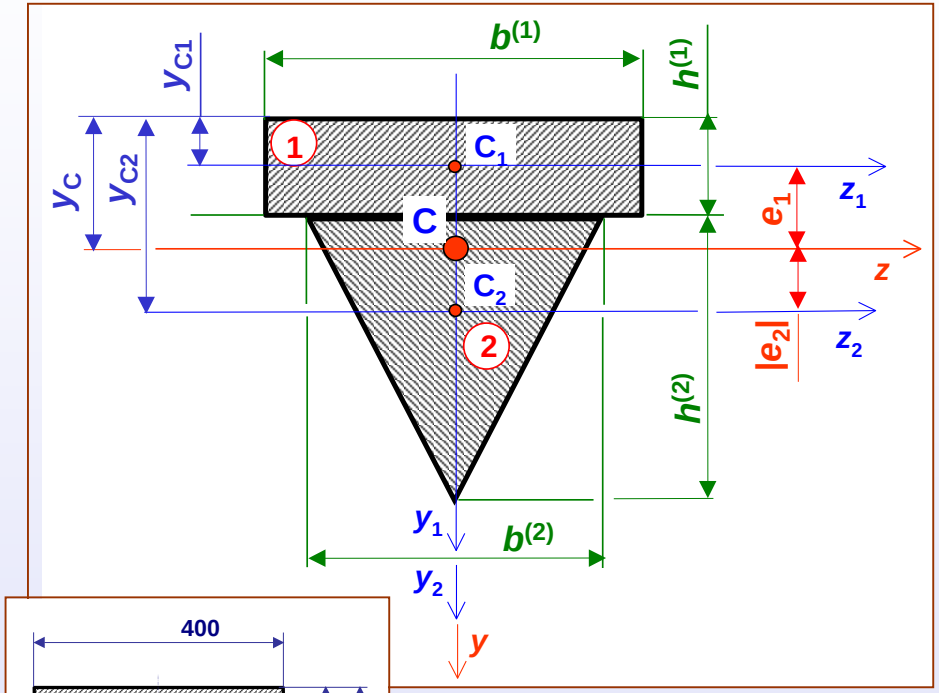
Pindala $A^{(1)}$ või $A^{(2)}$

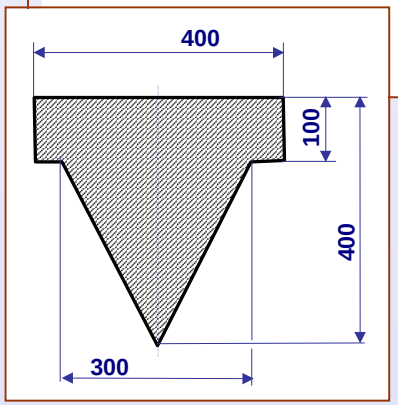
Rööpse telje M koordinaat K suhtes e_1 või e_2

$$e_1 = y_C - y_{C1} = 129 - 50 = 79 \text{ mm}$$

$$e_2 = y_C - y_{C2} = 129 - 200 = -71 \text{ mm}$$

4.3.1. Ristlõike inertsimoment telje *z* suhtes (2)





Osakujundite inertsimomendid

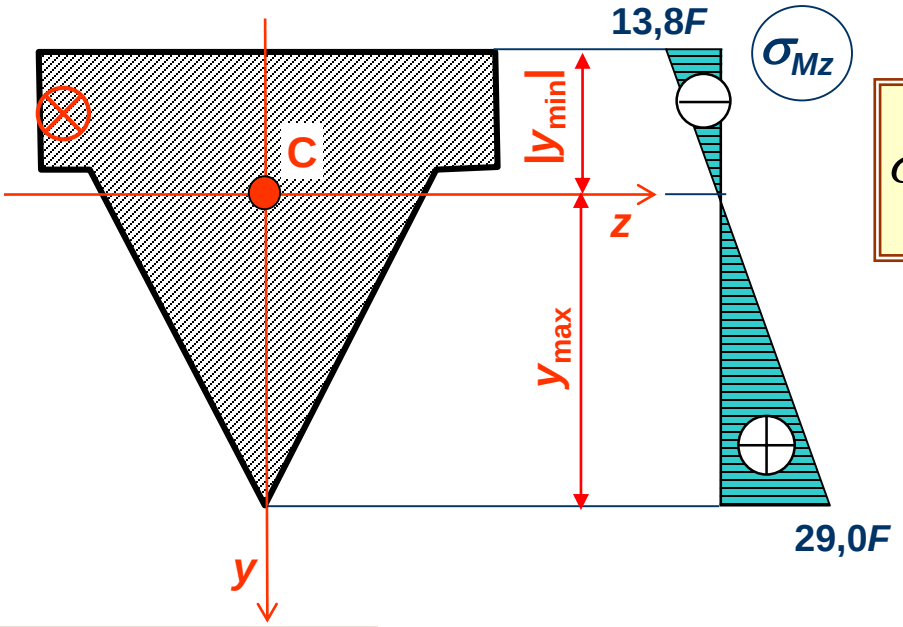
$$I_z^{(1)} = I_{z1}^{(1)} + e_1^2 A^{(1)} = \frac{b^{(1)} h^{(1)3}}{12} + e_1^2 A^{(1)} =$$
$$= \frac{400 \cdot 100^3}{12} + 79^2 \cdot 40000 =$$
$$= 282{,}9 \cdot 10^6 \text{ mm}^4 \approx 2{,}83 \cdot 10^{-4} \text{ m}^4$$

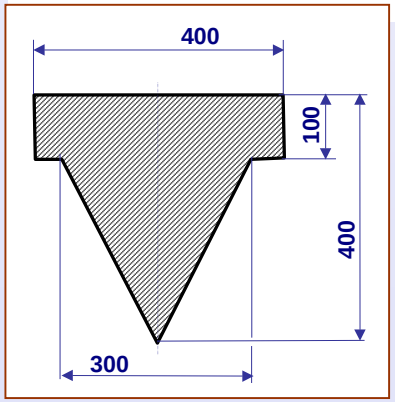
$$I_z^{(2)} = I_{z2}^{(2)} + e_2^2 A^{(2)} = \frac{b^{(2)} h^{(2)3}}{36} + e_2^2 A^{(2)} =$$
$$= \frac{300 \cdot 300^3}{36} + (-71)^2 \cdot 45000 =$$
$$= 451{,}8 \cdot 10^6 \text{ mm}^4 \approx 4{,}52 \cdot 10^{-4} \text{ m}^4$$

Liitkujundi inertsimoment

$$I_z = I_z^{(1)} + I_z^{(2)} = 2{,}83 \cdot 10^{-4} + 4{,}52 \cdot 10^{-4} = \underline{7{,}35 \cdot 10^{-4} \text{ m}^4}$$

4.3.2. Ristlõike paindepinge σ_{Mz} funktsioon





Ristlõike paindepinge σ_{Mz} laotus

$$\sigma_{Mz} = \frac{M_z}{I_z} y = \frac{0{,}079F}{7{,}35 \cdot 10^{-4}} y = 107{,}4Fy \approx 107Fy$$

Paindepinge funktsioon toe ristlõikes

$y_{max} = 400 - y_C = 400 - 129 = 271$ mm $= 0{,}271$ m

$y_{min} = - y_C = - 129$ mm $= - 0{,}129$ m

See on tõmbepinge (+)

See on survepinge (-)

Ristlõike suurimad paindepinged

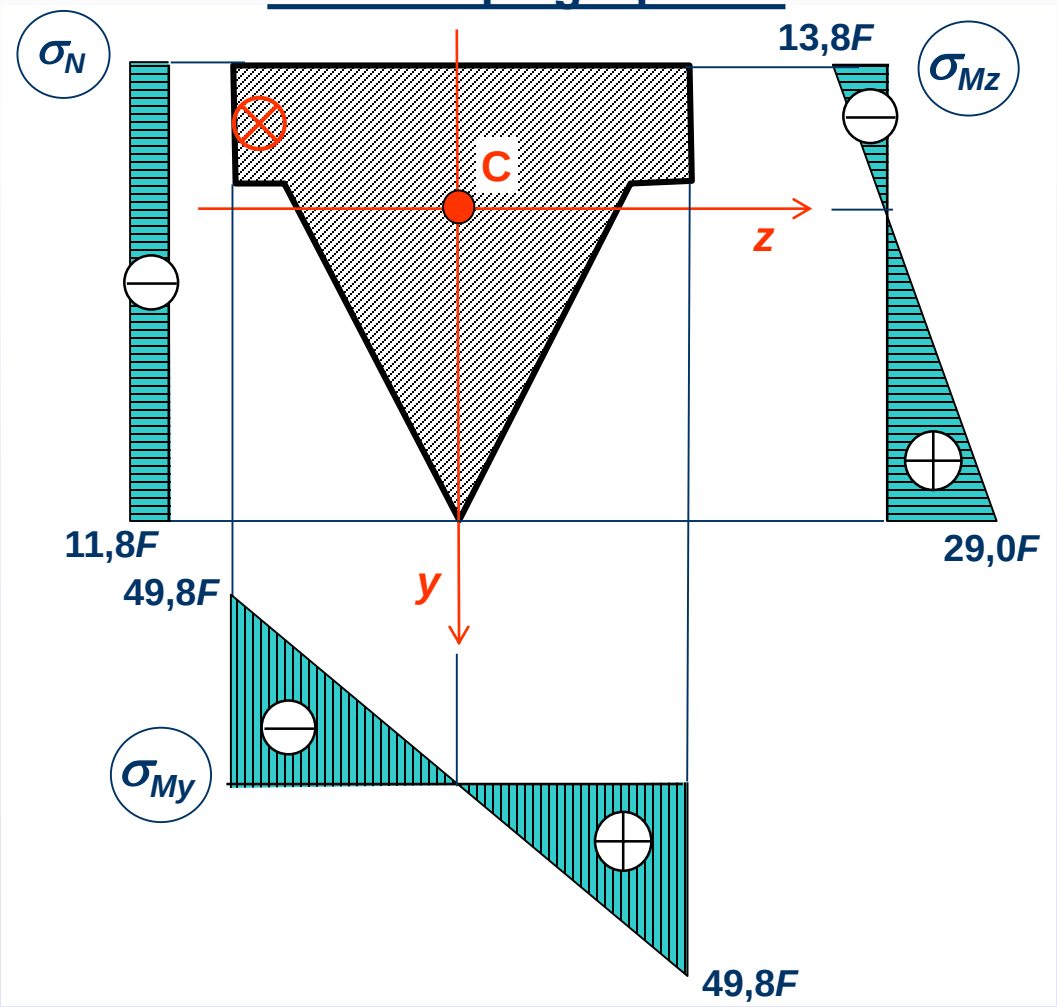
$$\sigma_{Mz}^{max} = 107Fy_{max} = 107 \cdot F \cdot 0{,}271 = 28{,}99F \approx 29{,}0F$$

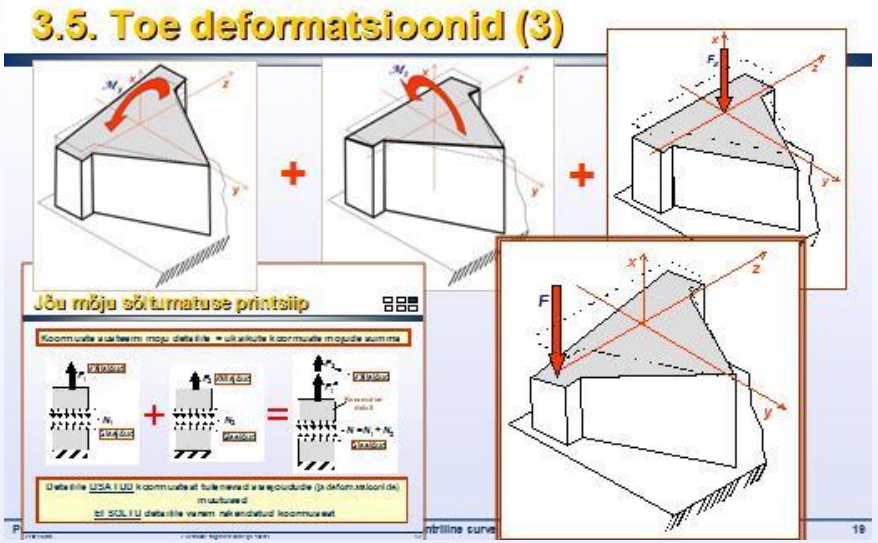
$$\sigma_{Mz}^{min} = 107Fy_{min} = 107 \cdot F \cdot (-0{,}129) = -13{,}80F \approx -13{,}8F$$

5. Toe ristlõike pingete analüüs

5.1. Ristlõike pingete koosmõju

Ristlõike pingeepüürid

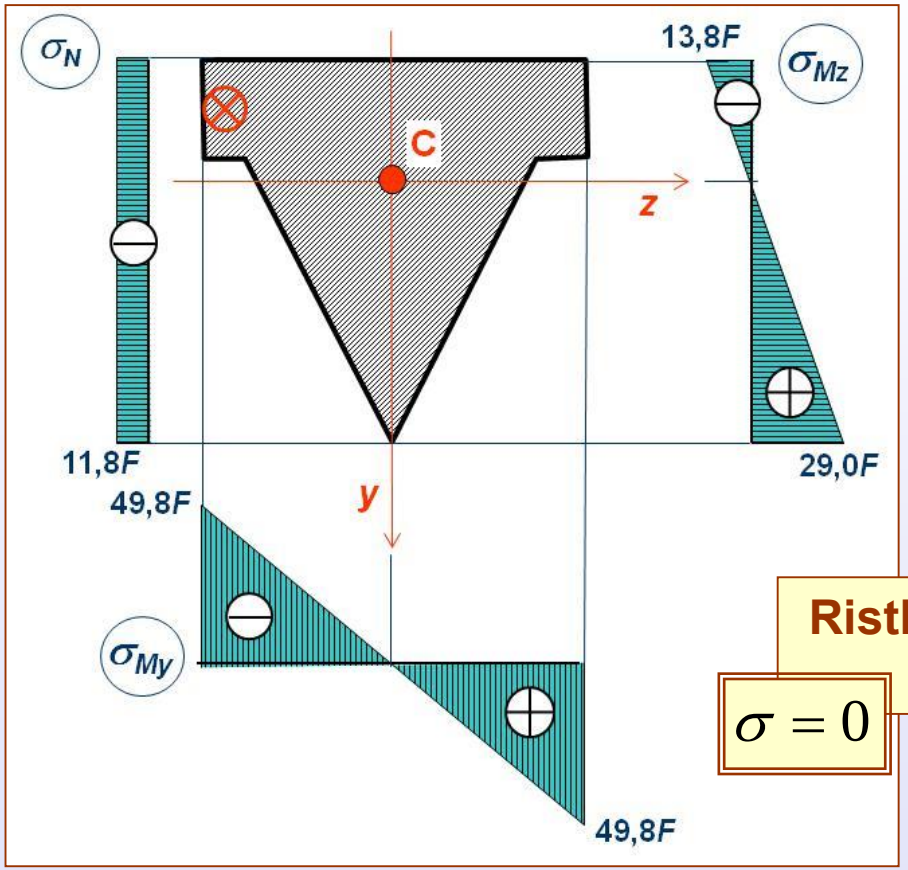




Ristlõike kõikide punktide deformatsioonid on telje *x* sihilised ja liituvad/lahutuvad

Ristlõike kõikide punktide pinged on telje *x* sihilised (normaalpinged) ja liituvad/lahutuvad

5.2. Ristlõike normaalpinge võrrand



Ristlõike punkti (*y*;*z*) summaarne normaalpinge

$$\sigma = \sigma_N + \sigma_{My} + \sigma_{Mz} = \\ = -11{,}8F + 249Fz + 107Fy$$

See on ristlõike punkti *(koordinaatidega y ja z)* ***normaalpinge funktsioon***

Ristlõike NULLJOON = joon ristlõike pinnal, mille punktide normaalpinge väärtus on NULL

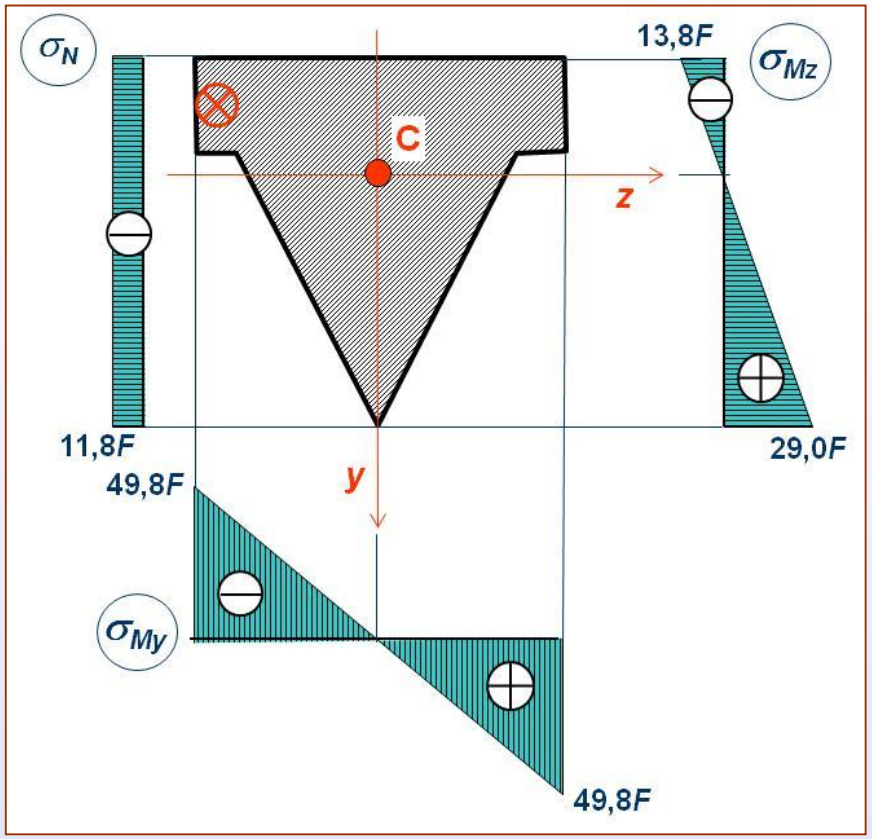
$$\sigma = 0$$

Ristlõike nulljoone võrrand

$$-11{,}8F + 249Fz + 107Fy = 0 \Rightarrow 249z + 107y = 11{,}8$$

*See on **SIRGE** võrrand*

5.3. Ristlõike ohtlikud punktid (1)



Ristlõike suurim tõmbepinge

$$\sigma = -11{,}8F + 249Fz + 107Fy = \max$$

Ristlõike suurim survepinge

$$\sigma = -11{,}8F + 249Fz + 107Fy = \min$$

PROBLEEM: Teada ei ole ohtlike punktide asukohad (koordinaadid *y* ja *z*)

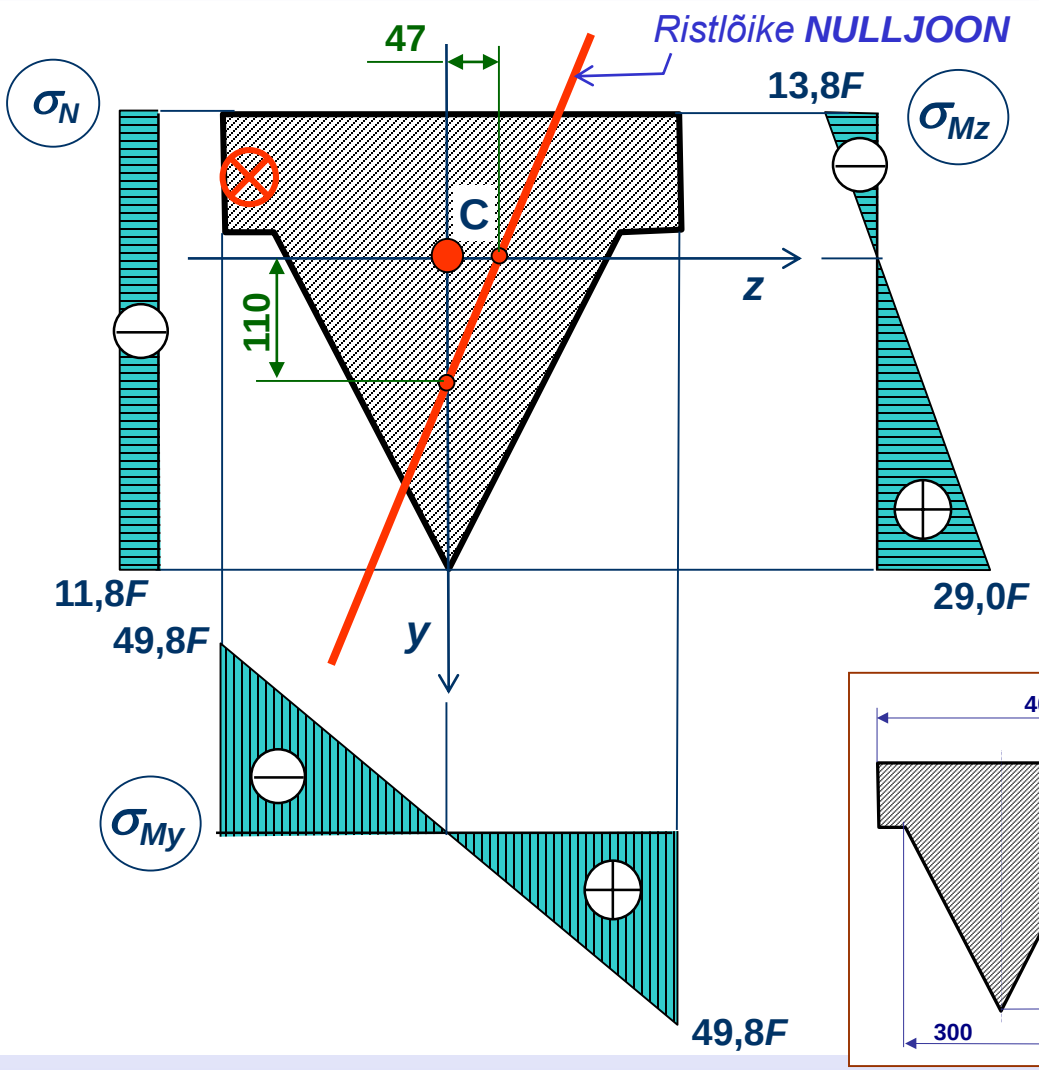
EELDUSED: Ristlõiked jäävad deformeerumisel tasapinnalisteks

Ristlõike nulljoon on sirge

JÄRELDUSED: Suurimad pinged (ja deformatsioonid) **on nulljoonest kõige kaugemates punktides**

Suurim tõmbepinge on nulljoonest ühel ja suurim survepinge teisel pool

5.3. Ristlõike ohtlikud punktid (2)



Ohtlike punktide paiknemise saab määrta geomeetriliselt

Nulljoone võrrand

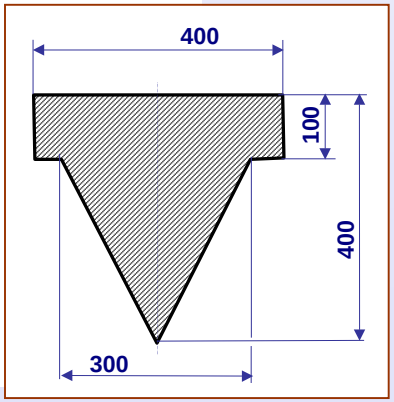
$$249z + 107y = 11{,}8$$

***NB!** Need on **koordinaadid** (märgiga + või -)*

Punktid, kus nulljoon lõikab telgi

Kui *y* = 0, siis: $z = \frac{11{,}8}{249} = 0{,}0473\ \text{m} \approx 47\ \text{mm}$

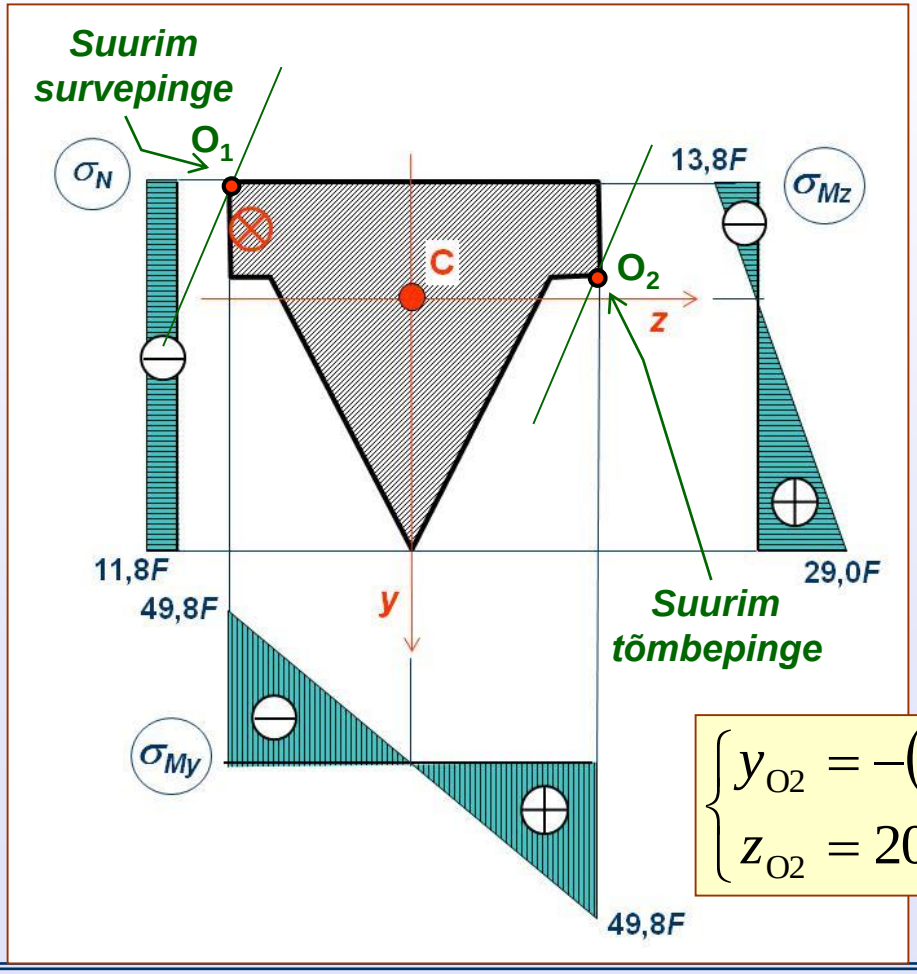
Kui *z* = 0, siis: $y = \frac{11{,}8}{107} = 0{,}1102\ \text{m} \approx 110\ \text{mm}$



NB! Ristlõike NULLJOON paikneb nii, et pinnakese C jääb koormuse *F* mõjusirge ja nulljoone vahele

5.3. Ristlõike ohtlikud punktid (3)

Ohtlike punktide asukohad määratakse nulljoonega paralleelsete sigete abil



Nulljoonest kõige kaugemal asetsevad ristlõike punktid

Punkt O_1 koordinaatidega:

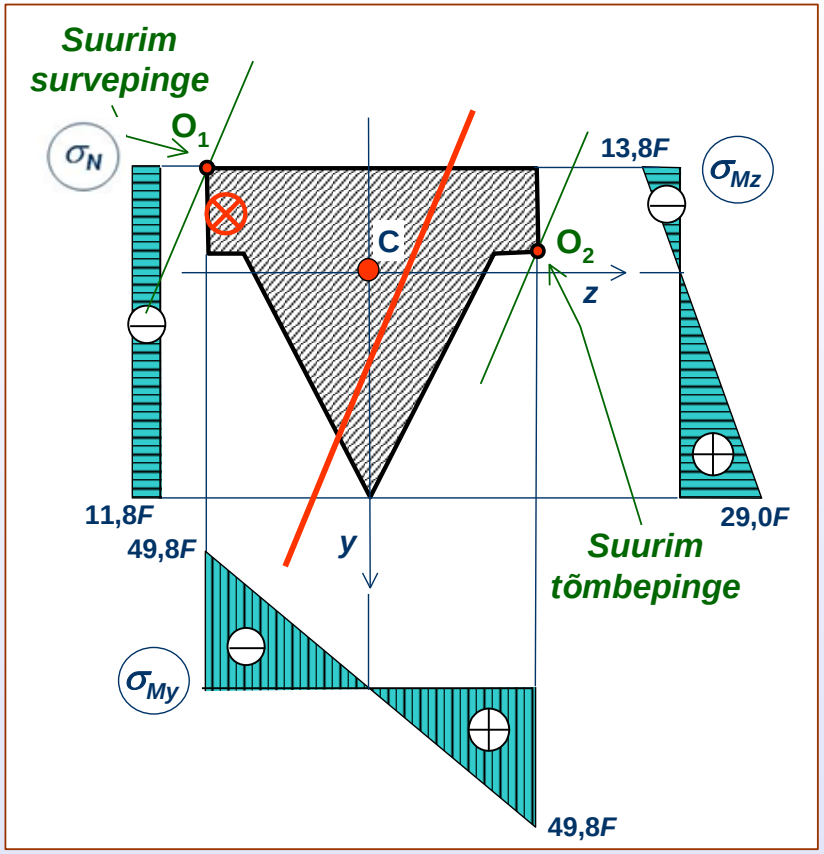
$$\begin{cases} y_{O1} = -y_C = -129\ \text{mm} = -0{,}129\ \text{m} \\ z_{O1} = -200\ \text{mm} = -0{,}2\ \text{m} \end{cases}$$

Siin on SURVEPINGE, kuna koormus F on SURUV

Punkt O_2 koordinaatidega:

$$\begin{cases} y_{O2} = -(y_C - 100) = -(129 - 100) = -29\ \text{mm} = -0{,}029\ \text{m} \\ z_{O2} = 200\ \text{mm} = 0{,}2\ \text{m} \end{cases}$$

5.4. Ristlõike suurimad pinged



Ristlõike suurim survepinge

$$\sigma_{O1} = -11{,}8F + 249Fz_{O1} + 107Fy_{O1} =$$
$$= -11{,}8 \cdot F + 249 \cdot F \cdot (-0{,}2) + 107 \cdot F \cdot (-0{,}129) =$$
$$= -75{,}40F \approx \underline{-75{,}4F}$$

KONTROLL: σ_{O1} PIDI olema survepinge → σ_{O1} on märgiga "–", järelikult ON survepinge

Ristlõike suurim tõmbepinge

$$\sigma_{O2} = -11{,}8F + 249Fz_{O2} + 107Fy_{O2} =$$
$$= -11{,}8 \cdot F + 249 \cdot F \cdot 0{,}2 + 107 \cdot F \cdot (-0{,}029) =$$
$$= 34{,}89F \approx \underline{34{,}9F}$$

KONTROLL: Kui σ_{O1} on survepinge, siis σ_{O2} PEAB olema tõmbepinge → σ_{O2} on märgiga "+", järelikult ON tõmbepinge

6. Tugevusarvutus

6.1. Toele lubatav koormus

Materjal: MALM
Lubatav survepinge: $[\sigma]_{Surve}$ = 200 MPa
Lubatav tõmbepinge: $[\sigma]_{Tõmme}$ = 40 MPa

SURVE tugevustingimus ohtlikus punktis O_1

$$|\sigma_{O1}| \le [\sigma]_{Surve} \Rightarrow |-75{,}4F| \le 200 \cdot 10^6 \Rightarrow F \le \frac{200 \cdot 10^6}{75{,}4} = 2{,}65 \cdot 10^6 \text{ N} \approx 2600 \text{ kN}$$

Punkti O_1 jaoks ohutu koormus *F*

TÕMBE tugevustingimus ohtlikus punktis O_2

$$\sigma_{O2} \le [\sigma]_{Tõmme} \Rightarrow 34{,}9F \le 40 \cdot 10^6 \Rightarrow F \le \frac{40 \cdot 10^6}{34{,}9} = 1{,}14 \cdot 10^6 \text{ N} \approx 1100 \text{ kN}$$

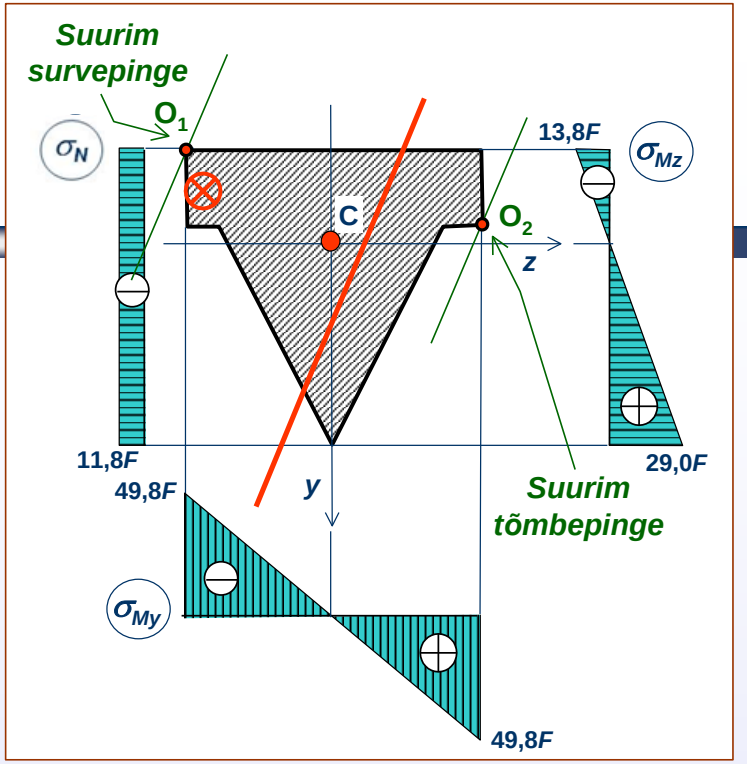
Punkti O_2 jaoks ohutu koormus *F*

Toele lubatav koormus:

$$F = \min(2600 \text{ kN}; 1100 \text{ kN}) = 1100 \text{ kN}$$

See koormuse väärtus on ohutu mõlema ohtliku punkti jaoks

6.2. Tugevuskontroll



Materjal: MALM
Lubatav survepinge: $[\sigma]_{Surve}$ = 200 MPa
Lubatav tõmbepinge: $[\sigma]_{Tõmme}$ = 40 MPa

Ohtlikus SURVEpunktis O_1

$$\left|\sigma_{O1}\right| = \left|-75{,}4F\right| = 75{,}4 \cdot 1100 \cdot 10^3 = 82{,}9 \cdot 10^6 \text{ Pa} \approx 83 \text{ MPa} \leq 200 \text{ MPa}$$

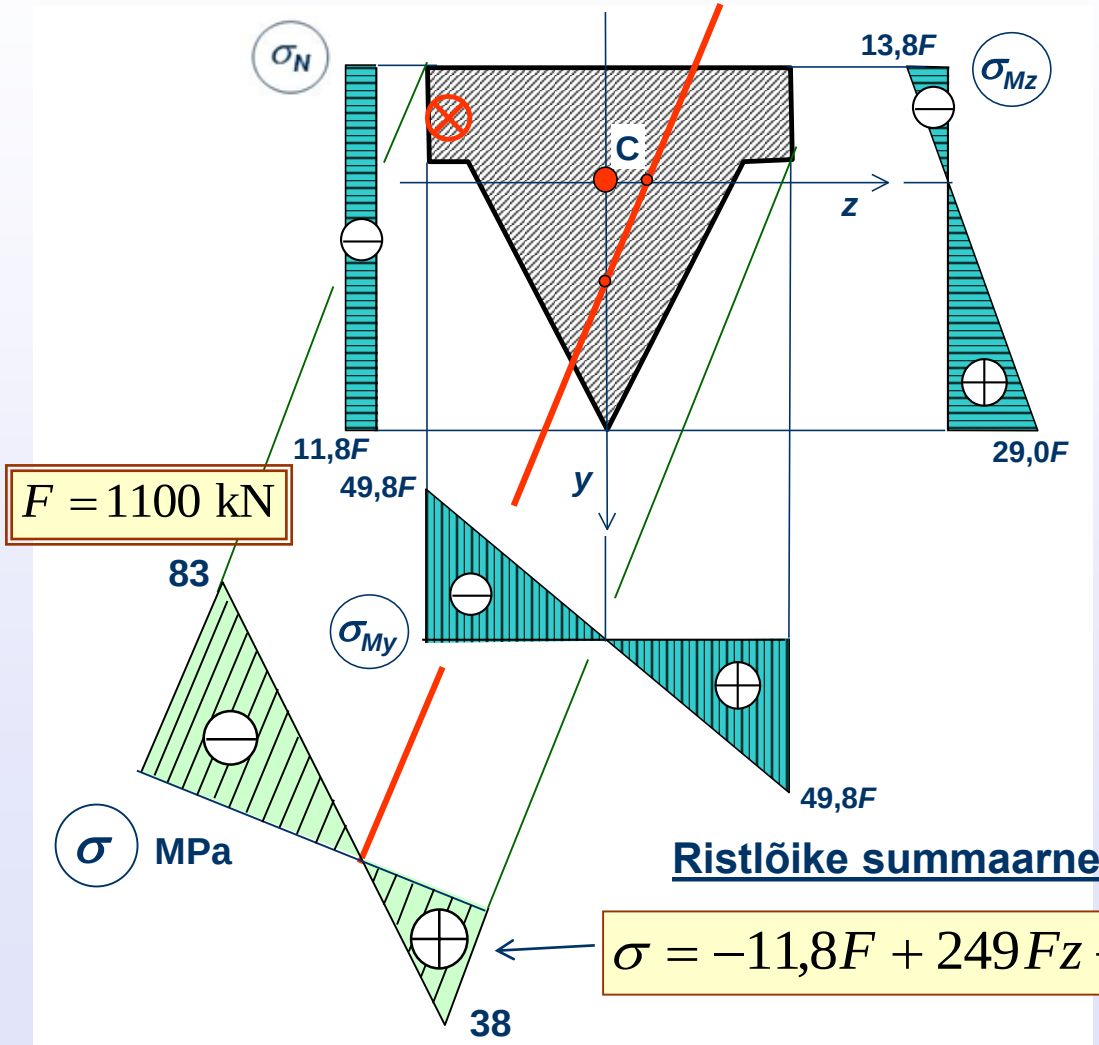
Ohtlikus survepunktis on tugevus tagatud

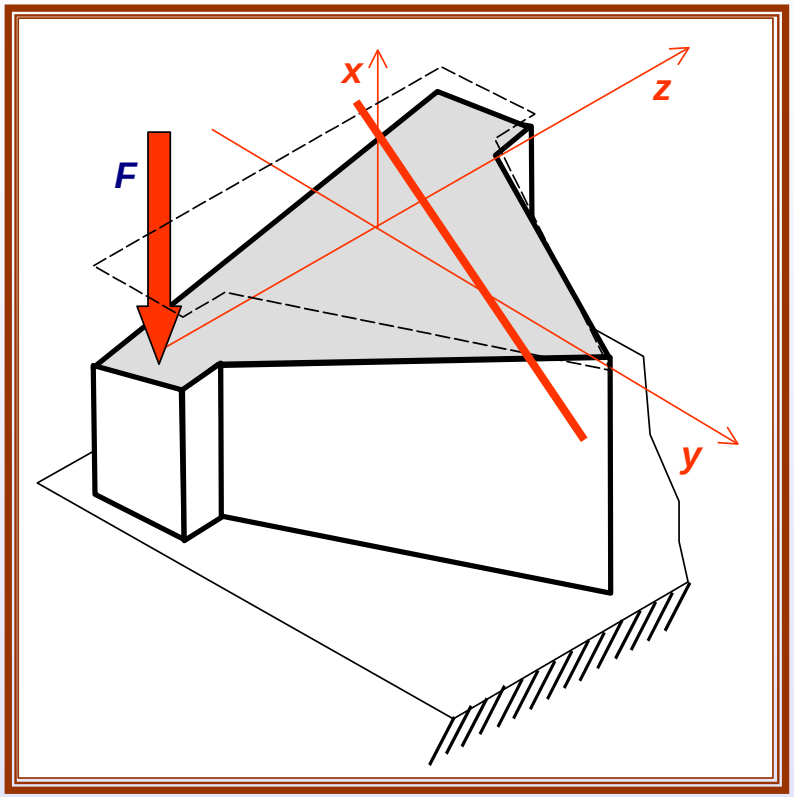
Ohtlikus TÕMBEpunktis O_2

$$\sigma_{O2} = 34{,}9F = 34{,}9 \cdot 1100 \cdot 10^3 = 38{,}3 \cdot 10^6 \text{ Pa} \approx 38 \text{ MPa} \leq 40 \text{ MPa}$$

Ohtlikus tõmbepunktis on tugevus tagatud

6.3. Summaarse pinge epüür





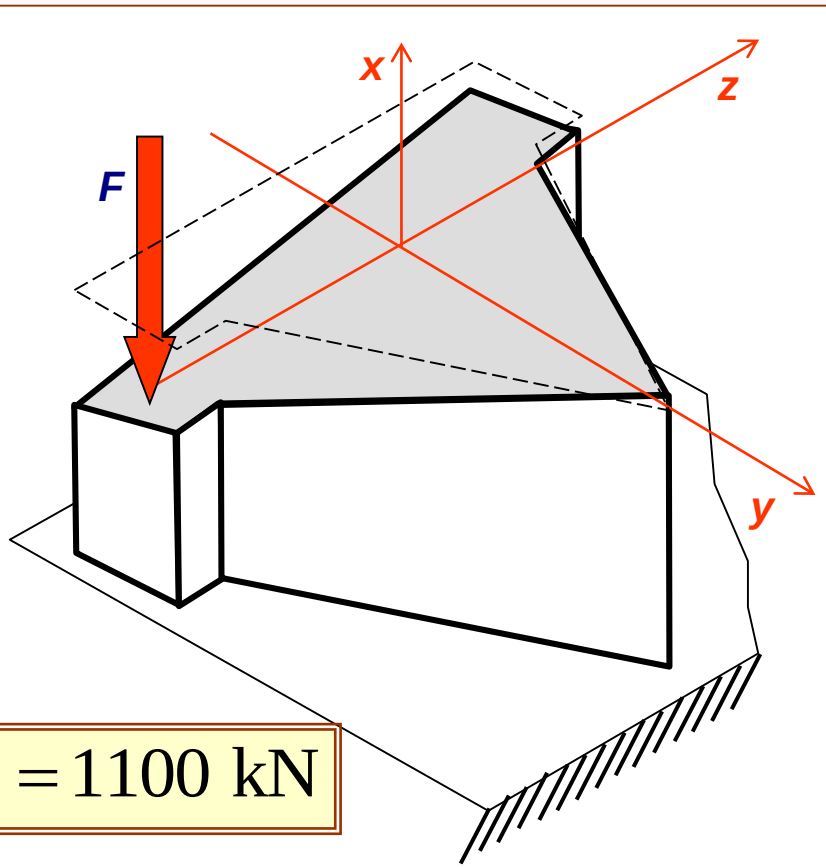
Ristlõike summaarne pinge

$$\sigma = -11{,}8F + 249Fz + 107Fy$$

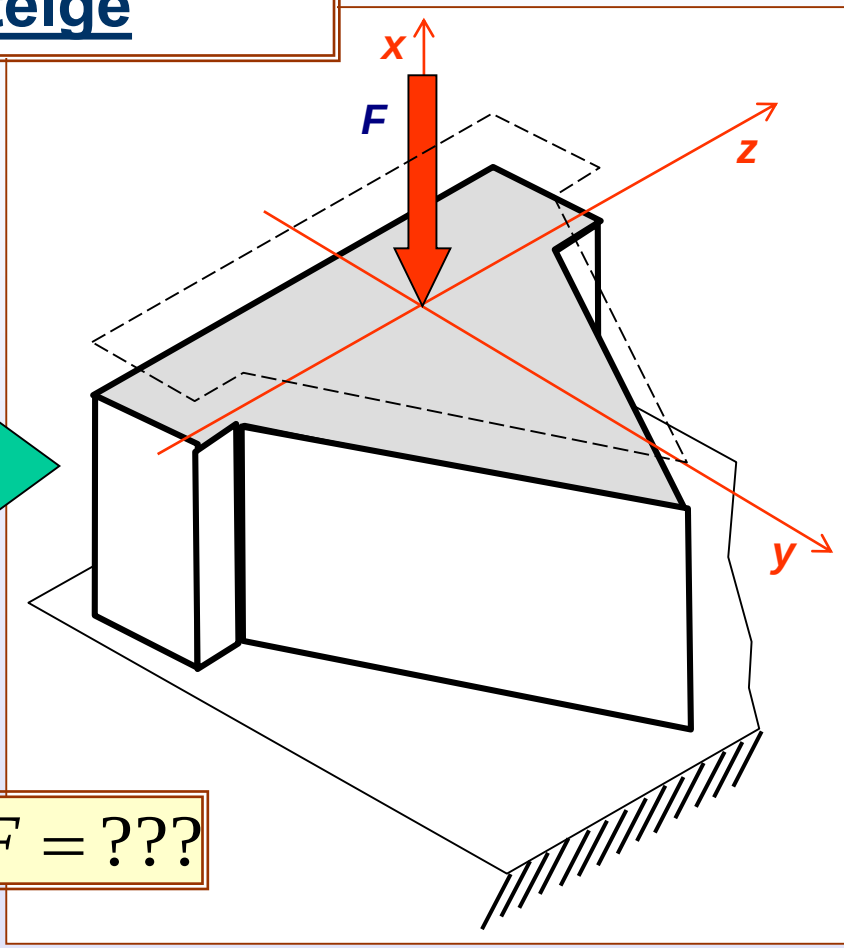
7. Jätkuülesanne

7.1. Jätkuülesande püstitus

Kui suur on toe lubatava koormuse väärtus, kui koormus mõjub piki toe telge



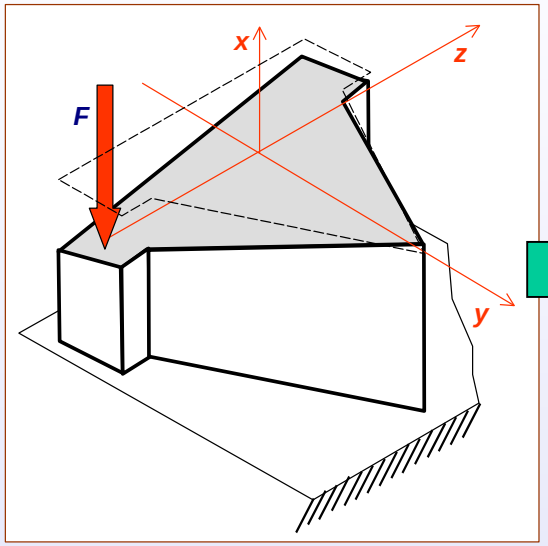
$F = 1100\ \text{kN}$



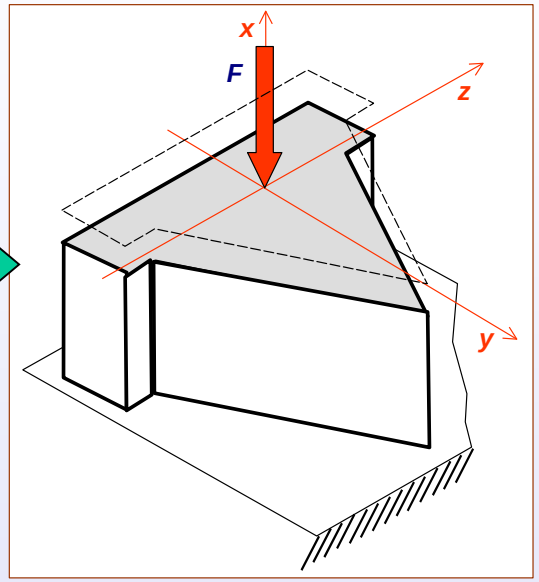
$F = ???$

7.2. Tugevusarvutus

Ekstsentriline pike



Pike (surve)



Pikke tugevustingimus

$$|\sigma| \le [\sigma]_{\text{Surve}}$$

Pikke SURVEpinge

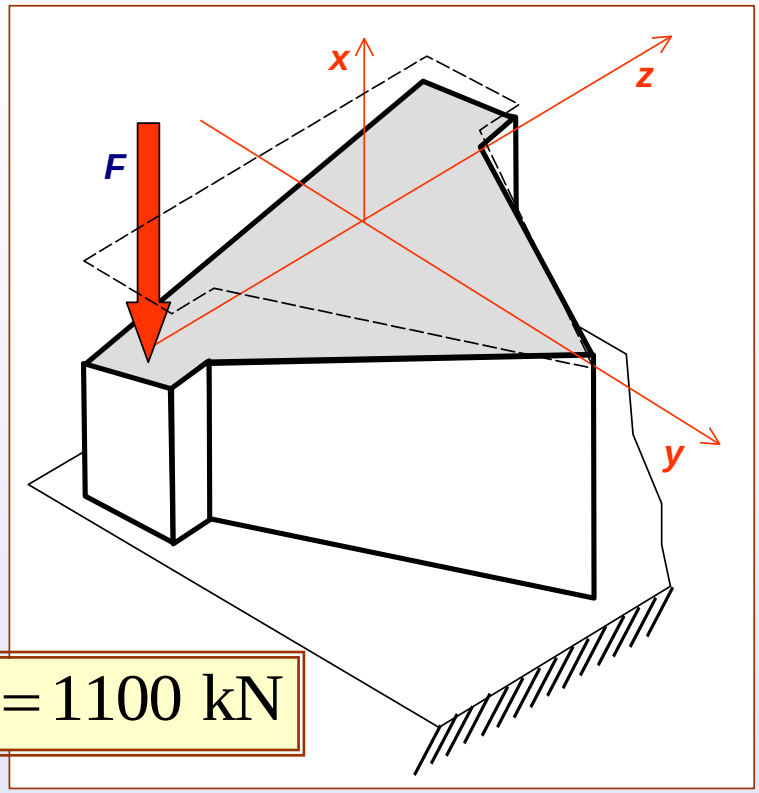
$$\sigma_N = \frac{N}{A} = \frac{-F}{A^{(1)} + A^{(2)}} = \frac{-F}{40000 \cdot 10^{-6} + 45000 \cdot 10^{-6}} = -11{,}76F \approx -11{,}8F$$

$$|-11{,}8F| \le 200 \cdot 10^6$$

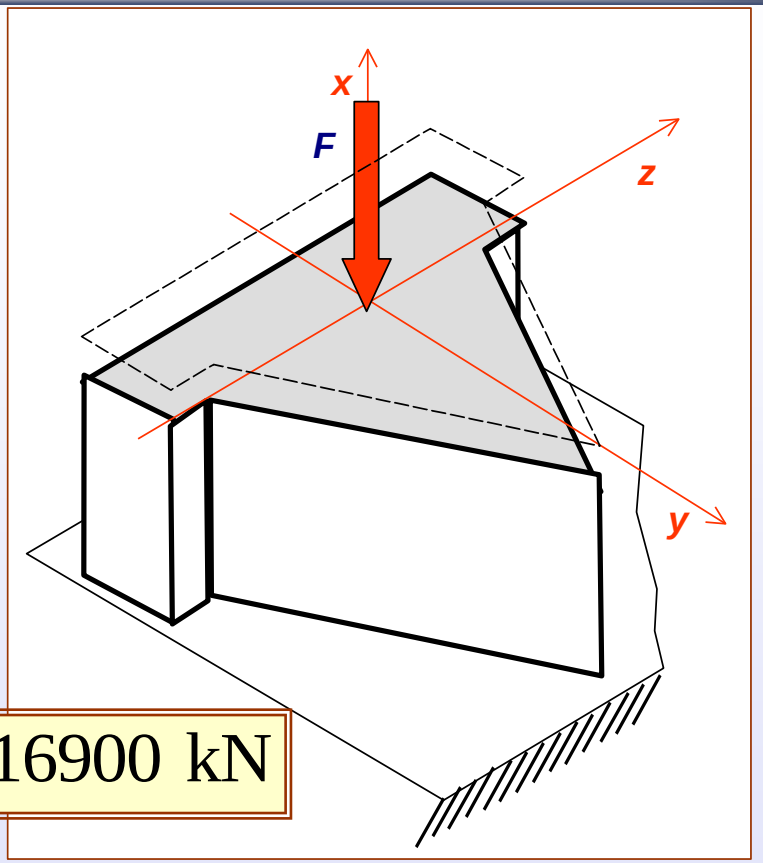
Materjal: MALM
Lubatav survepinge: $[\sigma]_{\text{Surve}}$ = 200 MPa
Lubatav tõmbepinge: $[\sigma]_{\text{Tõmme}}$ = 40 MPa

$$F \le \frac{200 \cdot 10^6}{11{,}8} = 16{,}94 \cdot 10^6 \text{ N} \approx 16900 \text{ kN}$$

7.3. Tugede võrdlus ja oluline järeldus



$F = 1100\ \text{kN}$



$F = 16900\ \text{kN}$

Pikikoormuse ekstsentrilisus on OHTLIK

Mida suurem on pikikoormuse ekstsentrilisus, seda väiksem on detaili kandevõime
(selles ülesandes on erinevus 15 x)