

Net Present Value and Other Investment Criteria

A company's shareholders prefer to be rich rather than poor. Therefore, they want the firm to invest in every project that is worth more than it costs. The difference between a project's value and its cost is its *net present value (NPV)*. Companies can best help their shareholders by investing in all projects with a positive NPV and rejecting those with a negative NPV.

We start this chapter with a review of the net present value rule. We then turn to some other measures that companies may look at when making investment decisions. The first two of these measures, the project's payback period and its book rate of return, are little better than rules of thumb, easy to calculate and easy to communicate. Although there is a place for rules of thumb in this world, an engineer needs something more accurate when designing a 100-story building, and a financial manager needs more than a rule of thumb when making a substantial capital investment decision.

Instead of calculating a project's NPV, companies often compare the expected rate of return from investing in the project with the return that shareholders could earn on equivalent-risk investments in the capital market. The company accepts those projects that provide a higher return than shareholders could earn for themselves. If used correctly, this rate of return rule should always identify projects that increase firm value. However, we shall see that the rule sets several traps for the unwary.

We conclude the chapter by showing how to cope with situations when the firm has only limited capital. This raises two problems. One is computational. In simple cases we just choose those projects that give the highest NPV per dollar invested, but more elaborate techniques are sometimes needed to sort through the possible alternatives. The other problem is to decide whether capital rationing really exists and whether it invalidates the net present value rule. Guess what? NPV, properly interpreted, wins out in the end.

5-1 A Review of the Basics

Vegetron's chief financial officer (CFO) is wondering how to analyze a proposed \$1 million investment in a new venture called project X. He asks what you think.

Your response should be as follows: "First, forecast the cash flows generated by project X over its economic life. Second, determine the appropriate opportunity cost of capital (r). This should reflect both the time value of money and the risk involved in project X. Third, use this opportunity cost of capital to discount the project's future cash flows. The sum of the discounted cash flows is called present value (PV). Fourth, calculate *net present value (NPV)* by subtracting the \$1 million investment from PV. If we call the cash flows C_0 , C_1 , and so on, then

$$\text{NPV} = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \cdots$$

We should invest in project X if its NPV is greater than zero.”

However, Vegetron’s CFO is unmoved by your sagacity. He asks why NPV is so important.

Your reply: “Let us look at what is best for Vegetron stockholders.” They want you to make their Vegetron shares as valuable as possible.

“Right now Vegetron’s total market value (price per share times the number of shares outstanding) is \$10 million. That includes \$1 million cash we can invest in project X.” The value of Vegetron’s other assets and opportunities must therefore be \$9 million. We have to decide whether it is better to keep the \$1 million cash and reject project X or to spend the cash and accept the project. Let us call the value of the new project PV. Then the choice is as follows:

Asset	Market Value (\$ millions)	
	Reject Project X	Accept Project X
Cash	1	0
Other assets	9	9
Project X	0	PV
	10	9 + PV

“Clearly project X is worthwhile if its present value, PV, is greater than \$1 million, that is, if net present value is positive.”

CFO: “How do I know that the PV of project X will actually show up in Vegetron’s market value?”

Your reply: “Suppose we set up a new, independent firm X, whose only asset is project X.” What would be the market value of firm X?

“Investors would forecast the dividends that firm X would pay and discount those dividends by the expected rate of return of securities having similar risks.” We know that stock prices are equal to the present value of forecasted dividends.

“Since project X is the only asset, the dividend payments we would expect firm X to pay are exactly the cash flows we have forecasted for project X.” Moreover, the rate investors would use to discount firm X’s dividends is exactly the rate we should use to discount project X’s cash flows.

“I agree that firm X is entirely hypothetical.” But if project X is accepted, investors holding Vegetron stock will really hold a portfolio of project X and the firm’s other assets. We know the other assets are worth \$9 million considered as a separate venture. Since asset values add up, we can easily figure out the portfolio value once we calculate the value of project X as a separate venture.

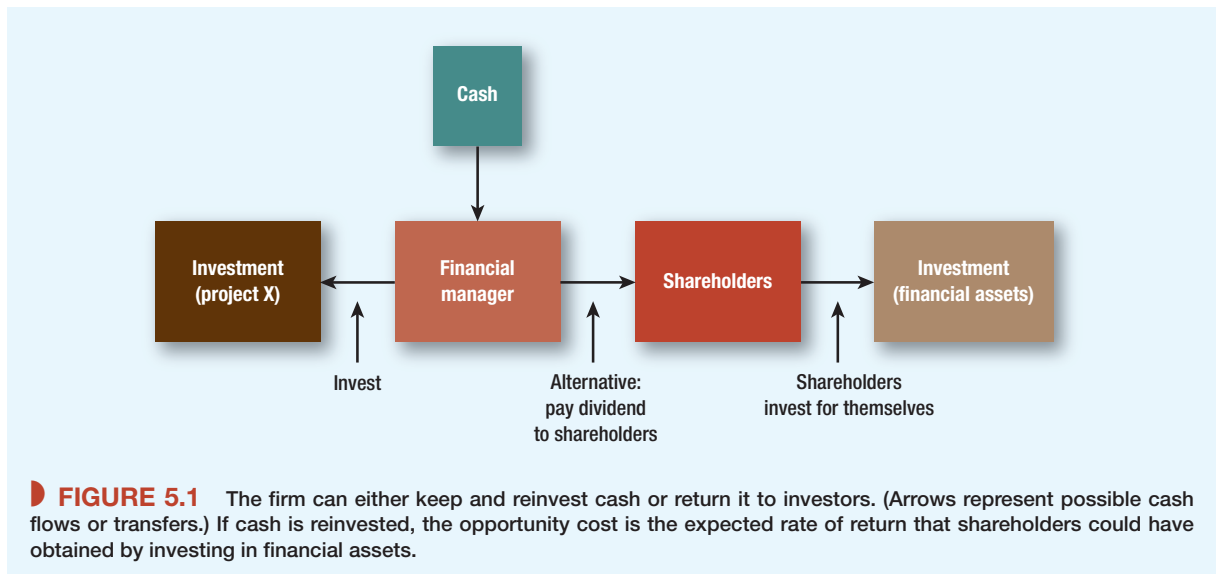
“By calculating the present value of project X, we are replicating the process by which the common stock of firm X would be valued in capital markets.”

CFO: “The one thing I don’t understand is where the discount rate comes from.”

Your reply: “I agree that the discount rate is difficult to measure precisely.” But it is easy to see what we are *trying* to measure. The discount rate is the opportunity cost of investing in the project rather than in the capital market. In other words, instead of accepting a project, the firm can always return the cash to the shareholders and let them invest it in financial assets.

“You can see the trade-off (Figure 5.1). The opportunity cost of taking the project is the return shareholders could have earned had they invested the funds on their own. When we discount the project’s cash flows by the expected rate of return on financial assets, we are measuring how much investors would be prepared to pay for your project.”

“But which financial assets?” Vegetron’s CFO queries. “The fact that investors expect only 12% on IBM stock does not mean that we should purchase Fly-by-Night Electronics if it offers 13%.”



Your reply: “The opportunity-cost concept makes sense only if assets of equivalent risk are compared. In general, you should identify financial assets that have the same risk as your project, estimate the expected rate of return on these assets, and use this rate as the opportunity cost.”

Net Present Value's Competitors

When you advised the CFO to calculate the project's NPV, you were in good company. These days 75% of firms always, or almost always, calculate net present value when deciding on investment projects. However, as you can see from Figure 5.2, NPV is not the only investment criterion that companies use, and firms often look at more than one measure of a project's attractiveness.

About three-quarters of firms calculate the project's internal rate of return (or IRR); that is roughly the same proportion as use NPV. The IRR rule is a close relative of NPV and, when used properly, it will give the same answer. You therefore need to understand the IRR rule and how to take care when using it.

A large part of this chapter is concerned with explaining the IRR rule, but first we look at two other measures of a project's attractiveness—the project's payback and its book rate of return. As we will explain, both measures have obvious defects. Few companies rely on them to make their investment decisions, but they do use them as supplementary measures that may help to distinguish the marginal project from the no-brainer.

Later in the chapter we also come across one further investment measure, the profitability index. Figure 5.2 shows that it is not often used, but you will find that there are circumstances in which this measure has some special advantages.

Three Points to Remember about NPV

As we look at these alternative criteria, it is worth keeping in mind the following key features of the net present value rule. First, the NPV rule recognizes that *a dollar today is worth more than a dollar tomorrow*, because the dollar today can be invested to start earning interest immediately. Any investment rule that does not recognize the *time value of money* cannot be sensible. Second, net present value depends solely on the *forecasted cash flows* from the project and the *opportunity cost of capital*. Any investment rule that is affected by the manager's tastes, the company's choice of accounting method, the profitability of the

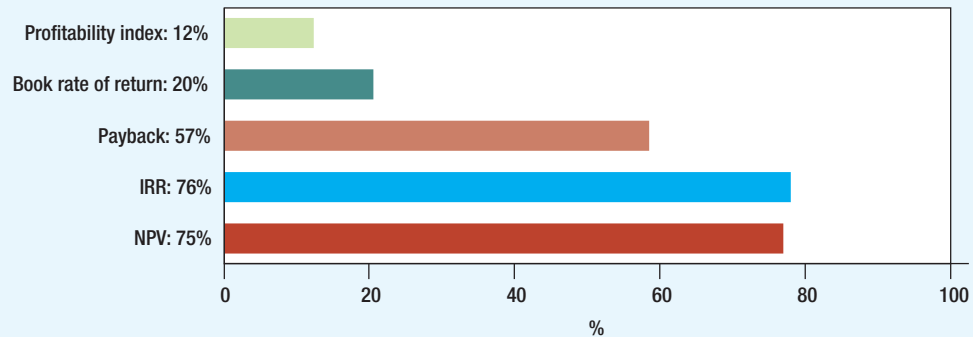


FIGURE 5.2 Survey evidence on the percentage of CFOs who always, or almost always, use a particular technique for evaluating investment projects.

Source: Reprinted from J. R. Graham and C. R. Harvey, "The Theory and Practice of Finance: Evidence from the Field," *Journal of Financial Economics* 61 (2001), pp. 187–243, © 2001 with permission from Elsevier Science.

company's existing business, or the profitability of other independent projects will lead to inferior decisions. Third, *because present values are all measured in today's dollars, you can add them up*. Therefore, if you have two projects A and B, the net present value of the combined investment is

$$\text{NPV}(A + B) = \text{NPV}(A) + \text{NPV}(B)$$

This adding-up property has important implications. Suppose project B has a negative NPV. If you tack it onto project A, the joint project (A + B) must have a lower NPV than A on its own. Therefore, you are unlikely to be misled into accepting a poor project (B) just because it is packaged with a good one (A). As we shall see, the alternative measures do not have this property. If you are not careful, you may be tricked into deciding that a package of a good and a bad project is better than the good project on its own.

NPV Depends on Cash Flow, Not on Book Returns

Net present value depends only on the project's cash flows and the opportunity cost of capital. But when companies report to shareholders, they do not simply show the cash flows. They also report book—that is, accounting—income and book assets.

Financial managers sometimes use these numbers to calculate a book (or accounting) rate of return on a proposed investment. In other words, they look at the prospective book income as a proportion of the book value of the assets that the firm is proposing to acquire:

$$\text{Book rate of return} = \frac{\text{book income}}{\text{book assets}}$$

Cash flows and book income are often very different. For example, the accountant labels some cash outflows as *capital investments* and others as *operating expenses*. The operating expenses are, of course, deducted immediately from each year's income. The capital expenditures are put on the firm's balance sheet and then depreciated. The annual depreciation charge is deducted from each year's income. Thus the book rate of return depends on which items the accountant treats as capital investments and how rapidly they are depreciated.¹

¹This chapter's mini-case contains simple illustrations of how book rates of return are calculated and of the difference between accounting income and project cash flow. Read the case if you wish to refresh your understanding of these topics. Better still, do the case calculations.

Now the merits of an investment project do not depend on how accountants classify the cash flows² and few companies these days make investment decisions just on the basis of the book rate of return. But managers know that the company's shareholders pay considerable attention to book measures of profitability and naturally they think (and worry) about how major projects would affect the company's book return. Those projects that would reduce the company's book return may be scrutinized more carefully by senior management.

You can see the dangers here. The company's book rate of return may not be a good measure of true profitability. It is also an *average* across all of the firm's activities. The average profitability of past investments is not usually the right hurdle for new investments. Think of a firm that has been exceptionally lucky and successful. Say its average book return is 24%, double shareholders' 12% opportunity cost of capital. Should it demand that all *new* investments offer 24% or better? Clearly not: That would mean passing up many positive-NPV opportunities with rates of return between 12 and 24%.

We will come back to the book rate of return in Chapters 12 and 28, when we look more closely at accounting measures of financial performance.

5-2 Payback

We suspect that you have often heard conversations that go something like this: "We are spending \$6 a week, or around \$300 a year, at the laundromat. If we bought a washing machine for \$800, it would pay for itself within three years. That's well worth it." You have just encountered the payback rule.

A project's **payback period** is found by counting the number of years it takes before the cumulative cash flow equals the initial investment. For the washing machine the payback period was just under three years. The **payback rule** states that a project should be accepted if its payback period is less than some specified cutoff period. For example, if the cutoff period is four years, the washing machine makes the grade; if the cutoff is two years, it doesn't.

EXAMPLE 5.1 • The Payback Rule

Consider the following three projects:

Project	Cash Flows, \$				Payback Period (years)	NPV at 10%
	C_0	C_1	C_2	C_3		
A	-2,000	500	500	5,000	3	+2,624
B	-2,000	500	1,800	0	2	-58
C	-2,000	1,800	500	0	2	+50

Project A involves an initial investment of \$2,000 ($C_0 = -2,000$) followed by cash inflows during the next three years. Suppose the opportunity cost of capital is 10%. Then project A has an NPV of +\$2,624:

$$\text{NPV(A)} = -2,000 + \frac{500}{1.10} + \frac{500}{1.10^2} + \frac{5,000}{1.10^3} = +\$2,624$$

²Of course, the depreciation method used for tax purposes does have cash consequences that should be taken into account in calculating NPV. We cover depreciation and taxes in the next chapter.

Project B also requires an initial investment of \$2,000 but produces a cash inflow of \$500 in year 1 and \$1,800 in year 2. At a 10% opportunity cost of capital project B has an NPV of $-\$58$:

$$\text{NPV(B)} = -2,000 + \frac{500}{1.10} + \frac{1,800}{1.10^2} = -\$58$$

The third project, C, involves the same initial outlay as the other two projects but its first-period cash flow is larger. It has an NPV of $+\$50$.

$$\text{NPV(C)} = -2,000 + \frac{1,800}{1.10} + \frac{500}{1.10^2} = +\$50$$

The net present value rule tells us to accept projects A and C but to reject project B.

Now look at how rapidly each project pays back its initial investment. With project A you take three years to recover the \$2,000 investment; with projects B and C you take only two years. If the firm used the *payback rule* with a cutoff period of two years, it would accept only projects B and C; if it used the payback rule with a cutoff period of three or more years, it would accept all three projects. Therefore, regardless of the choice of cutoff period, the payback rule gives different answers from the net present value rule.

You can see why payback can give misleading answers as illustrated in Example 5.1:

1. *The payback rule ignores all cash flows after the cutoff date.* If the cutoff date is two years, the payback rule rejects project A regardless of the size of the cash inflow in year 3.
2. *The payback rule gives equal weight to all cash flows before the cutoff date.* The payback rule says that projects B and C are equally attractive, but because C's cash inflows occur earlier, C has the higher net present value at any discount rate.

To use the payback rule, a firm must decide on an appropriate cutoff date. If it uses the same cutoff regardless of project life, it will tend to accept many poor short-lived projects and reject many good long-lived ones.

We have had little good to say about the payback rule. So why do many companies continue to use it? Senior managers don't truly believe that all cash flows after the payback period are irrelevant. We suggest three explanations. First, payback may be used because it is the simplest way to *communicate* an idea of project profitability. Investment decisions require discussion and negotiation among people from all parts of the firm, and it is important to have a measure that everyone can understand. Second, managers of larger corporations may opt for projects with short paybacks because they believe that quicker profits mean quicker promotion. That takes us back to Chapter 1 where we discussed the need to align the objectives of managers with those of shareholders. Finally, owners of family firms with limited access to capital may worry about their future ability to raise capital. These worries may lead them to favor rapid payback projects even though a longer-term venture may have a higher NPV.

Discounted Payback

Occasionally companies discount the cash flows before they compute the payback period. The discounted cash flows for our three projects are as follows:

Discounted Cash Flows, \$						
Project	C_0	C_1	C_2	C_3	Discounted Payback Period (years)	NPV at 20%
A	-2,000	$500/1.10 = 455$	$500/1.10^2 = 413$	$5,000/1.10^3 = 3,757$	3	+2,624
B	-2,000	$500/1.10 = 455$	$1,800/1.10^2 = 1,488$		—	-58
C	-2,000	$1,800/1.10 = 1,636$	$500/1.10^2 = 413$		2	+50

The *discounted payback rule* asks, How many years does the project have to last in order for it to make sense in terms of net present value? You can see that the value of the cash inflows from project B never exceeds the initial outlay and would always be rejected under the discounted payback rule. Thus the discounted payback rule will never accept a negative-NPV project. On the other hand, it still takes no account of cash flows after the cutoff date, so that good long-term projects such as A continue to risk rejection.

Rather than automatically rejecting any project with a long discounted payback period, many managers simply use the measure as a warning signal. These managers don't unthinkingly reject a project with a long discounted-payback period. Instead they check that the proposer is not unduly optimistic about the project's ability to generate cash flows into the distant future. They satisfy themselves that the equipment has a long life and that competitors will not enter the market and eat into the project's cash flows.

5-3 Internal (or Discounted-Cash-Flow) Rate of Return

Whereas payback and return on book are ad hoc measures, internal rate of return has a much more respectable ancestry and is recommended in many finance texts. If, therefore, we dwell more on its deficiencies, it is not because they are more numerous but because they are less obvious.

In Chapter 2 we noted that the net present value rule could also be expressed in terms of rate of return, which would lead to the following rule: "Accept investment opportunities offering rates of return in excess of their opportunity costs of capital." That statement, properly interpreted, is absolutely correct. However, interpretation is not always easy for long-lived investment projects.

There is no ambiguity in defining the true rate of return of an investment that generates a single payoff after one period:

$$\text{Rate of return} = \frac{\text{payoff}}{\text{investment}} - 1$$

Alternatively, we could write down the NPV of the investment and find the discount rate that makes NPV = 0.

$$\text{NPV} = C_0 + \frac{C_1}{1 + \text{discount rate}} = 0$$

implies

$$\text{Discount rate} = \frac{C_1}{-C_0} - 1$$

Of course C_1 is the payoff and $-C_0$ is the required investment, and so our two equations say exactly the same thing. *The discount rate that makes $NPV = 0$ is also the rate of return.*

How do we calculate return when the project produces cash flows in several periods? Answer: we use the same definition that we just developed for one-period projects—the *project rate of return is the discount rate that gives a zero NPV*. This discount rate is known as the **discounted-cash-flow (DCF) rate of return** or **internal rate of return (IRR)**. The internal rate of return is used frequently in finance. It can be a handy measure, but, as we shall see, it can also be a misleading measure. You should, therefore, know how to calculate it and how to use it properly.

Calculating the IRR

The internal rate of return is defined as the rate of discount that makes $NPV = 0$. So to find the IRR for an investment project lasting T years, we must solve for IRR in the following expression:

$$NPV = C_0 + \frac{C_1}{1 + \text{IRR}} + \frac{C_2}{(1 + \text{IRR})^2} + \cdots + \frac{C_T}{(1 + \text{IRR})^T} = 0$$

Actual calculation of IRR usually involves trial and error. For example, consider a project that produces the following flows:

Cash Flows, \$		
C_0	C_1	C_2
-4,000	+2,000	+4,000

The internal rate of return is IRR in the equation

$$NPV = -4,000 + \frac{2,000}{1 + \text{IRR}} + \frac{4,000}{(1 + \text{IRR})^2} = 0$$

Let us arbitrarily try a zero discount rate. In this case NPV is not zero but +\$2,000:

$$NPV = -4,000 + \frac{2,000}{1.0} + \frac{4,000}{(1.0)^2} = +\$2,000$$

The NPV is positive; therefore, the IRR must be greater than zero. The next step might be to try a discount rate of 50%. In this case net present value is -\$889:

$$NPV = -4,000 + \frac{2,000}{1.50} + \frac{4,000}{(1.50)^2} = -\$889$$

The NPV is negative; therefore, the IRR must be less than 50%. In Figure 5.3 we have plotted the net present values implied by a range of discount rates. From this we can see that a discount rate of 28% gives the desired net present value of zero. Therefore IRR is 28%.

The easiest way to calculate IRR, if you have to do it by hand, is to plot three or four combinations of NPV and discount rate on a graph like Figure 5.3, connect the points with a smooth line, and read off the discount rate at which $NPV = 0$. It is of course quicker and more accurate to use a computer spreadsheet or a specially programmed calculator, and in practice this is what financial managers do. The *Useful Spreadsheet Functions* box near the end of the chapter presents Excel functions for doing so.

Some people confuse the internal rate of return and the opportunity cost of capital because both appear as discount rates in the NPV formula. The internal rate of return is a *profitability measure* that depends solely on the amount and timing of the project cash flows. The opportunity cost of capital is a *standard of profitability* that we use to calculate how much the project is worth. The opportunity cost of capital is established in capital markets. It is the expected rate of return offered by other assets with the same risk as the project being evaluated.

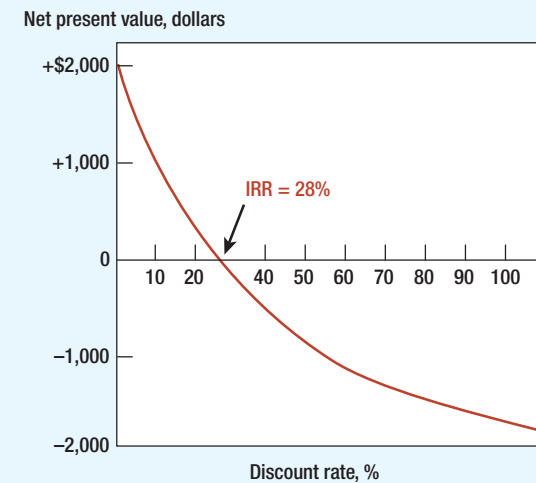


FIGURE 5.3

This project costs \$4,000 and then produces cash inflows of \$2,000 in year 1 and \$4,000 in year 2. Its internal rate of return (IRR) is 28%, the rate of discount at which NPV is zero.

The IRR Rule

The *internal rate of return rule* is to accept an investment project if the opportunity cost of capital is less than the internal rate of return. You can see the reasoning behind this idea if you look again at Figure 5.3. If the opportunity cost of capital is less than the 28% IRR, then the project has a *positive* NPV when discounted at the opportunity cost of capital. If it is equal to the IRR, the project has a *zero* NPV. And if it is greater than the IRR, the project has a *negative* NPV. Therefore, when we compare the opportunity cost of capital with the IRR on our project, we are effectively asking whether our project has a positive NPV. This is true not only for our example. The rule will give the same answer as the net present value rule *whenever the NPV of a project is a smoothly declining function of the discount rate*.

Many firms use internal rate of return as a criterion in preference to net present value. We think that this is a pity. Although, properly stated, the two criteria are formally equivalent, the internal rate of return rule contains several pitfalls.

Pitfall 1—Lending or Borrowing?

Not all cash-flow streams have NPVs that decline as the discount rate increases. Consider the following projects A and B:

Cash Flows, \$				
Project	C_0	C_1	IRR	NPV at 10%
A	-1,000	+1,500	+50%	+364
B	+1,000	-1,500	+50%	-364

Each project has an IRR of 50%. (In other words, $-1,000 + 1,500/1.50 = 0$ and $+1,000 - 1,500/1.50 = 0$.)

Does this mean that they are equally attractive? Clearly not, for in the case of A, where we are initially paying out \$1,000, we are *lending* money at 50%; in the case of B, where we are initially receiving \$1,000, we are *borrowing* money at 50%. When we lend money, we want a *high* rate of return; when we borrow money, we want a *low* rate of return.

If you plot a graph like Figure 5.3 for project B, you will find that NPV increases as the discount rate increases. Obviously the internal rate of return rule, as we stated it above, won't work in this case; we have to look for an IRR *less* than the opportunity cost of capital.

Pitfall 2—Multiple Rates of Return

Helmsley Iron is proposing to develop a new strip mine in Western Australia. The mine involves an initial investment of A\$30 billion and is expected to produce a cash inflow of A\$10 billion a year for the next nine years. At the end of that time the company will incur A\$65 billion of cleanup costs. Thus the cash flows from the project are:

Cash Flows (billions of Australian dollars)				
C_0	C_1	...	C_9	C_{10}
-30	10		10	-65

Helmsley calculates the project's IRR and its NPV as follows:

IRR (%)	NPV at 10%
+3.50 and 19.54	\$A2.53 billion

Note that there are *two* discount rates that make $NPV = 0$. That is, *each* of the following statements holds:

$$NPV = -30 + \frac{10}{1.035} + \frac{10}{1.035^2} + \cdots + \frac{10}{1.035^9} - \frac{65}{1.035^{10}} = 0$$

$$NPV = -30 + \frac{10}{1.1954} + \frac{10}{1.1954^2} + \cdots + \frac{10}{1.1954^9} - \frac{65}{1.1954^{10}} = 0$$

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Try It! Figure 5.4:
Helmsley's multiple IRRs



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In other words, the investment has an IRR of both 3.50 *and* 19.54%. Figure 5.4 shows how this comes about. As the discount rate increases, NPV initially rises and then declines. The reason for this is the double change in the sign of the cash-flow stream. There can be as many internal rates of return for a project as there are changes in the sign of the cash flows.³

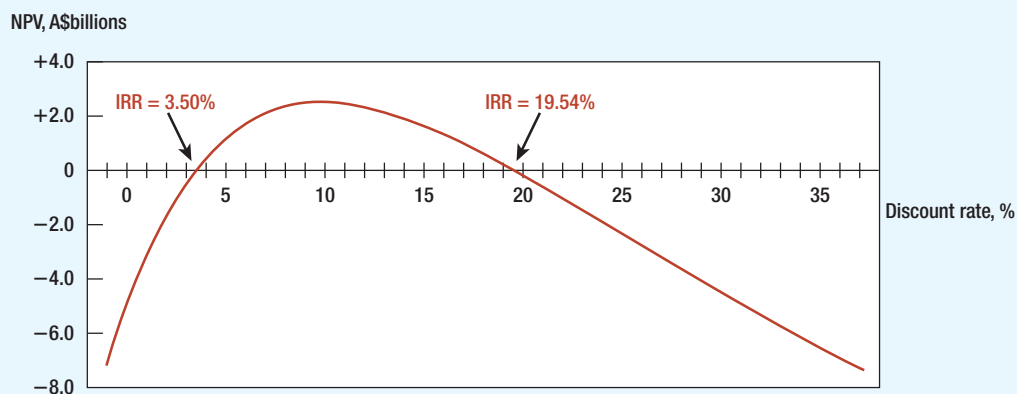


FIGURE 5.4 Helmsley Iron's mine has two internal rates of return. $NPV = 0$ when the discount rate is +3.50% and when it is +19.54%.

³By Descartes's "rule of signs" there can be as many different solutions to a polynomial as there are changes of sign.

Decommissioning and clean-up costs can sometimes be huge. Phillips Petroleum has estimated that it will need to spend \$1 billion to remove its Norwegian offshore oil platforms. It can cost over \$300 million to decommission a nuclear power plant. These are obvious instances where cash flows go from positive to negative, but you can probably think of a number of other cases where the company needs to plan for later expenditures. Ships periodically need to go into dry dock for a refit, hotels may receive a major face-lift, machine parts may need replacement, and so on.

Whenever the cash-flow stream is expected to change sign more than once, the company typically sees more than one IRR.

As if this is not difficult enough, there are also cases in which *no* internal rate of return exists. For example, project C has a positive net present value at all discount rates:

Cash Flows, \$					
Project	C_0	C_1	C_2	IRR (%)	NPV at 10%
C	+1,000	−3,000	+2,500	None	+339

A number of adaptations of the IRR rule have been devised for such cases. Not only are they inadequate, but they also are unnecessary, for the simple solution is to use net present value.⁴

Pitfall 3—Mutually Exclusive Projects

Firms often have to choose from among several alternative ways of doing the same job or using the same facility. In other words, they need to choose from among **mutually exclusive projects**. Here too the IRR rule can be misleading.

Consider projects D and E:

Cash Flows, \$				
Project	C_0	C_1	IRR (%)	NPV at 10%
D	−10,000	+20,000	100	+ 8,182
E	−20,000	+35,000	75	+11,818

Perhaps project D is a manually controlled machine tool and project E is the same tool with the addition of computer control. Both are good investments, but E has the higher NPV and is, therefore, better. However, the IRR rule seems to indicate that if you have to choose, you should go for D since it has the higher IRR. If you follow the IRR rule, you have the satisfaction of earning a 100% rate of return; if you follow the NPV rule, you are \$11,818 richer.

You can salvage the IRR rule in these cases by looking at the internal rate of return on the *incremental* flows. Here is how to do it: First, consider the smaller project (D in our example). It has an IRR of 100%, which is well in excess of the 10% opportunity cost of capital.

⁴Companies sometimes get around the problem of multiple rates of return by discounting the later cash flows back at the cost of capital until there remains only one change in the sign of the cash flows. A *modified internal rate of return* (MIRR) can then be calculated on this revised series. In our example, the MIRR is calculated as follows:

1. Calculate the present value in year 5 of all the subsequent cash flows:

$$PV \text{ in year 5} = 10/1.1 + 10/1.1^2 + 10/1.1^3 + 10/1.1^4 - 65/1.1^5 = -8.66$$

2. Add to the year 5 cash flow the present value of subsequent cash flows:

$$C_5 + PV(\text{subsequent cash flows}) = 10 - 8.66 = 1.34$$

3. Since there is now only one change in the sign of the cash flows, the revised series has a unique rate of return, which is 13.7%:

$$NPV = -30 + 10/1.137 + 10/1.137^2 + 10/1.137^3 + 10/1.137^4 + 1.34/1.137^5 = 0$$

Since the MIRR of 13.7% is greater than the cost of capital (and the initial cash flow is negative), the project has a positive NPV when valued at the cost of capital.

Of course, it would be much easier in such cases to abandon the IRR rule and just calculate project NPV.

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A project with no IRR



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You know, therefore, that D is acceptable. You now ask yourself whether it is worth making the additional \$10,000 investment in E. The incremental flows from undertaking E rather than D are as follows:

Cash Flows, \$				
Project	C_0	C_1	IRR (%)	NPV at 10%
E – D	–10,000	+15,000	50	+3,636

The IRR on the incremental investment is 50%, which is also well in excess of the 10% opportunity cost of capital. So you should prefer project E to project D.⁵

Unless you look at the incremental expenditure, IRR is unreliable in ranking projects of different scale. It is also unreliable in ranking projects that offer different patterns of cash flow over time. For example, suppose the firm can take project F *or* project G but not both:

Cash Flows, \$									
Project	C_0	C_1	C_2	C_3	C_4	C_5	Etc.	IRR (%)	NPV at 10%
F	–9,000	+6,000	+5,000	+4,000	0	0	...	33	3,592
G	–9,000	+1,800	+1,800	+1,800	+1,800	+1,800	...	20	9,000

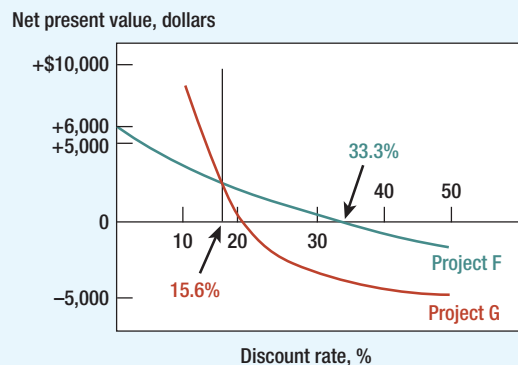
Project F has a higher IRR, but project G, which is a perpetuity, has the higher NPV. Figure 5.5 shows why the two rules give different answers. The green line gives the net present value of project F at different rates of discount. Since a discount rate of 33% produces a net present value of zero, this is the internal rate of return for project F. Similarly, the red line shows the net present value of project G at different discount rates. The IRR of project G is 20%. (We assume project G's cash flows continue indefinitely.) Note, however, that project G has a higher NPV as long as the opportunity cost of capital is less than 15.6%.

The reason that IRR is misleading is that the total cash inflow of project G is larger but tends to occur later. Therefore, when the discount rate is low, G has the higher NPV; when the discount rate is high, F has the higher NPV. (You can see from Figure 5.5 that the two projects have the *same* NPV when the discount rate is 15.6%.) The internal rates of return

on the two projects tell us that at a discount rate of 20% G has a zero NPV (IRR = 20%) and F has a positive NPV. Thus if the opportunity cost of capital were 20%, investors would place a higher value on the shorter-lived project F. But in our example the opportunity cost of capital is not 20% but 10%. So investors will pay a relatively high price for the longer-lived project. At a 10% cost of capital, an investment in G has an NPV of \$9,000 and an investment in F has an NPV of only \$3,592.⁶

FIGURE 5.5

The IRR of project F exceeds that of project G, but the NPV of project F is higher *only* if the discount rate is greater than 15.6%.



⁵You may, however, find that you have jumped out of the frying pan into the fire. The series of incremental cash flows may involve several changes in sign. In this case there are likely to be multiple IRRs and you will be forced to use the NPV rule after all.

⁶It is often suggested that the choice between the net present value rule and the internal rate of return rule should depend on the probable reinvestment rate. This is wrong. The prospective return on another *independent* investment should *never* be allowed to influence the investment decision.

This is a favorite example of ours. We have gotten many businesspeople's reactions to it. When asked to choose between F and G, many choose F. The reason seems to be the rapid payback generated by project F. In other words, they believe that if they take F, they will also be able to use the rapid cash inflows to make other investments in the future, whereas if they take G, they won't have money enough for these investments. In other words they implicitly assume that it is a *shortage of capital* that forces the choice between F and G. When this implicit assumption is brought out, they usually admit that G is better if there is no capital shortage.

But the introduction of capital constraints raises two further questions. The first stems from the fact that most of the executives preferring F to G work for firms that would have no difficulty raising more capital. Why would a manager at IBM, say, choose F on the grounds of limited capital? IBM can raise plenty of capital for future projects regardless of whether F or G is chosen; therefore these future opportunities should not affect the choice between F and G. The answer seems to be that large firms usually impose capital budgets on divisions and subdivisions as a part of the firm's planning and control system. Since the system is complicated and cumbersome, the budgets are not easily altered, and so they are perceived as real constraints by middle management.

The second question is this. If there is a capital constraint, either real or self-imposed, should IRR be used to rank projects? The answer is no. The problem in this case is to find the package of investment projects that satisfies the capital constraint and has the largest net present value. The IRR rule will not identify this package. As we will show in the next section, the only practical and general way to do so is to use the technique of linear programming.

When we have to choose between projects F and G, it is easiest to compare the net present values. But if your heart is set on the IRR rule, you can use it as long as you look at the internal rate of return on the incremental flows. The procedure is exactly the same as we showed above. First, you check that project F has a satisfactory IRR. Then you look at the return on the incremental cash flows from G.

Cash Flows, \$									
Project	C_0	C_1	C_2	C_3	C_4	C_5	Etc.	IRR (%)	NPV at 10%
G - F	0	-4,200	-3,200	-2,200	+1,800	+1,800	...	+15.6	+5,408

The IRR on the incremental cash flows from G is 15.6%. Since this is greater than the opportunity cost of capital, you should undertake G rather than F.⁷

Pitfall 4—What Happens When There Is More than One Opportunity Cost of Capital?

We have simplified our discussion of capital budgeting by assuming that the opportunity cost of capital is the same for all the cash flows, C_1 , C_2 , C_3 , etc. Remember our most general formula for calculating net present value:

$$NPV = C_0 + \frac{C_1}{1 + r_1} + \frac{C_2}{(1 + r_2)^2} + \frac{C_3}{(1 + r_3)^3} + \dots$$

In other words, we discount C_1 at the opportunity cost of capital for one year, C_2 at the opportunity cost of capital for two years, and so on. The IRR rule tells us to accept a project if the IRR is greater than the opportunity cost of capital. But what do we do when we have several opportunity costs? Do we compare IRR with r_1 , r_2 , r_3 , ...? Actually we would have to compute a complex weighted average of these rates to obtain a number comparable to IRR.

⁷Because F and G had the same 10% cost of capital, we could choose between the two projects by asking whether the IRR on the incremental cash flows was greater or less than 10%. But suppose that F and G had different risks and therefore different costs of capital. In that case there would be no simple yardstick for assessing whether the IRR on the incremental cash flows was adequate.

What does this mean for capital budgeting? It means trouble for the IRR rule whenever there is more than one opportunity cost of capital. Many firms use the IRR, thereby implicitly assuming that there is no difference between short-term and long-term discount rates. They do this for the same reason that we have so far finessed the issue: simplicity.⁸

The Verdict on IRR

We have given four examples of things that can go wrong with IRR. We spent much less space on payback or return on book. Does this mean that IRR is worse than the other two measures? Quite the contrary. There is little point in dwelling on the deficiencies of payback or return on book. They are clearly ad hoc measures that often lead to silly conclusions. The IRR rule has a much more respectable ancestry. It is less easy to use than NPV, but, used properly, it gives the same answer.

Nowadays few large corporations use the payback period or return on book as their primary measure of project attractiveness. Most use discounted cash flow or “DCF,” and for many companies DCF means IRR, not NPV. For “normal” investment projects with an initial cash outflow followed by a series of cash inflows, there is no difficulty in using the internal rate of return to make a simple accept/reject decision. However, we think that financial managers need to worry more about Pitfall 3. Financial managers never see all possible projects. Most projects are proposed by operating managers. A company that instructs nonfinancial managers to look first at project IRRs prompts a search for those projects with the highest IRRs rather than the highest NPVs. It also encourages managers to *modify* projects so that their IRRs are higher. Where do you typically find the highest IRRs? In short-lived projects requiring little up-front investment. Such projects may not add much to the value of the firm.

We don’t know why so many companies pay such close attention to the internal rate of return, but we suspect that it may reflect the fact that management does not trust the forecasts it receives. Suppose that two plant managers approach you with proposals for two new investments. Both have a positive NPV of \$1,400 at the company’s 8% cost of capital, but you nevertheless decide to accept project A and reject B. Are you being irrational?

The cash flows for the two projects and their NPVs are set out in the table below. You can see that, although both proposals have the same NPV, project A involves an investment of \$9,000, while B requires an investment of \$9 million. Investing \$9,000 to make \$1,400 is clearly an attractive proposition, and this shows up in A’s IRR of nearly 16%. Investing \$9 million to make \$1,400 might also be worth doing if you could be *sure* of the plant manager’s forecasts, but there is almost no room for error in project B. You could spend time and money checking the cash-flow forecasts, but is it really worth the effort? Most managers would look at the IRR and decide that, if the cost of capital is 8%, a project that offers a return of 8.01% is not worth the worrying time.

Alternatively, management may conclude that project A is a clear winner that is worth undertaking right away, but in the case of project B it may make sense to wait and see whether the decision looks more clear-cut in a year’s time.⁹ Management postpones the decision on projects such as B by setting a hurdle rate for the IRR that is higher than the cost of capital.

Cash Flows (\$ thousands)						
Project	C_0	C_1	C_2	C_3	NPV at 8%	IRR (%)
A	−9.0	2.9	4.0	5.4	1.4	15.58
B	−9,000	2,560	3,540	4,530	1.4	8.01

⁸In Chapter 9 we look at some other cases in which it would be misleading to use the same discount rate for both short-term and long-term cash flows.

⁹In Chapter 22 we discuss when it may pay a company to delay undertaking a positive-NPV project. We will see that when projects are “deep-in-the-money” (project A), it generally pays to invest right away and capture the cash flows. However, in the case of projects that are “close-to-the-money” (project B) it makes more sense to wait and see.

5-4 Choosing Capital Investments When Resources Are Limited

Our entire discussion of methods of capital budgeting has rested on the proposition that the wealth of a firm's shareholders is highest if the firm accepts *every* project that has a positive net present value. Suppose, however, that there are limitations on the investment program that prevent the company from undertaking all such projects. Economists call this *capital rationing*. When capital is rationed, we need a method of selecting the package of projects that is within the company's resources yet gives the highest possible net present value.

An Easy Problem in Capital Rationing

Let us start with a simple example. The opportunity cost of capital is 10%, and our company has the following opportunities:

Cash Flows (\$ millions)				
Project	C_0	C_1	C_2	NPV at 10%
A	-10	+30	+5	21
B	-5	+5	+20	16
C	-5	+5	+15	12

All three projects are attractive, but suppose that the firm is limited to spending \$10 million. In that case, it can invest *either* in project A *or* in projects B and C, but it cannot invest in all three. Although individually B and C have lower net present values than project A, when taken together they have the higher net present value. Here we cannot choose between projects solely on the basis of net present values. When funds are limited, we need to concentrate on getting the biggest bang for our buck. In other words, we must pick the projects that offer the highest net present value per dollar of initial outlay. This ratio is known as the **profitability index**.¹⁰

$$\text{Profitability index} = \frac{\text{net present value}}{\text{investment}}$$

For our three projects the profitability index is calculated as follows:¹¹

Project	Investment (\$ millions)	NPV (\$ millions)	Profitability Index
A	10	21	2.1
B	5	16	3.2
C	5	12	2.4

Project B has the highest profitability index and C has the next highest. Therefore, if our budget limit is \$10 million, we should accept these two projects.¹²

¹⁰If a project requires outlays in two or more periods, the denominator should be the present value of the outlays. A few companies do not discount the benefits or costs before calculating the profitability index. The less said about these companies the better.

¹¹Sometimes the profitability index is defined as the ratio of the present value to initial outlay, that is, as PV/investment. This measure is also known as the *benefit-cost ratio*. To calculate the benefit-cost ratio, simply add 1.0 to each profitability index. Project rankings are unchanged.

¹²If a project has a positive profitability index, it must also have a positive NPV. Therefore, firms sometimes use the profitability index to select projects when capital is *not* limited. However, like the IRR, the profitability index can be misleading when used to choose between mutually exclusive projects. For example, suppose you were forced to choose between (1) investing \$100 in a project whose payoffs have a present value of \$200 or (2) investing \$1 million in a project whose payoffs have a present value of \$1.5 million. The first investment has the higher profitability index; the second makes you richer.

Unfortunately, there are some limitations to this simple ranking method. One of the most serious is that it breaks down whenever more than one resource is rationed.¹³ For example, suppose that the firm can raise only \$10 million for investment in *each* of years 0 and 1 and that the menu of possible projects is expanded to include an investment next year in project D:

Cash Flows (\$ millions)					
Project	C_0	C_1	C_2	NPV at 10%	Profitability Index
A	-10	+30	+ 5	21	2.1
B	- 5	+ 5	+20	16	3.2
C	- 5	+ 5	+15	12	2.4
D	0	-40	+60	13	0.4

One strategy is to accept projects B and C; however, if we do this, we cannot also accept D, which costs more than our budget limit for period 1. An alternative is to accept project A in period 0. Although this has a lower net present value than the combination of B and C, it provides a \$30 million positive cash flow in period 1. When this is added to the \$10 million budget, we can also afford to undertake D next year. A and D have *lower* profitability indexes than B and C, but they have a *higher* total net present value.

The reason that ranking on the profitability index fails in this example is that resources are constrained in each of two periods. In fact, this ranking method is inadequate whenever there is *any* other constraint on the choice of projects. This means that it cannot cope with cases in which two projects are mutually exclusive or in which one project is dependent on another.

For example, suppose that you have a long menu of possible projects starting this year and next. There is a limit on how much you can invest in each year. Perhaps also you can't undertake both project alpha and beta (they both require the same piece of land), and you can't invest in project gamma unless you also invest in delta (gamma is simply an add-on to delta). You need to find the package of projects that satisfies all these constraints and gives the highest NPV.

One way to tackle such a problem is to work through all possible combinations of projects. For each combination you first check whether the projects satisfy the constraints and then calculate the net present value. But it is smarter to recognize that linear programming (LP) techniques are specially designed to search through such possible combinations.

Uses of Capital Rationing Models

Linear programming models seem tailor-made for solving capital budgeting problems when resources are limited. Why then are they not universally accepted either in theory or in practice? One reason is that these models can turn out to be very complex. Second, as with any sophisticated long-range planning tool, there is the general problem of getting good data. It is just not worth applying costly, sophisticated methods to poor data. Furthermore, these models are based on the assumption that all future investment opportunities are known. In reality, the discovery of investment ideas is an unfolding process.

Our most serious misgivings center on the basic assumption that capital is limited. When we come to discuss company financing, we shall see that most large corporations do not face capital rationing and can raise large sums of money on fair terms. Why then do many company presidents tell their subordinates that capital is limited? If they are right, the capital market is seriously imperfect. What then are they doing maximizing NPV?¹⁴ We might be tempted to suppose that if capital is not rationed, they do not *need* to use linear programming and, if it is

BEYOND THE PAGE

Capital rationing models



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¹³It may also break down if it causes some money to be left over. It might be better to spend all the available funds even if this involves accepting a project with a slightly lower profitability index.

¹⁴Don't forget that in the Appendix to Chapter 1 we had to assume perfect capital markets to derive the NPV rule.

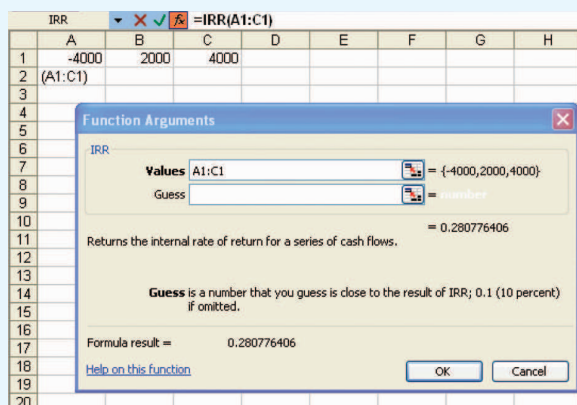
USEFUL SPREADSHEET FUNCTIONS

Internal Rate of Return

Spreadsheet programs such as Excel provide built-in functions to solve for internal rates of return. You can find these functions by pressing *fx* on the Excel toolbar. If you then click on the function that you wish to use, Excel will guide you through the inputs that are required. At the bottom left of the function box there is a Help facility with an example of how the function is used.

Here is a list of useful functions for calculating internal rates of return, together with some points to remember when entering data:

- **IRR:** Internal rate of return on a series of regularly spaced cash flows.
- **XIRR:** The same as IRR, but for irregularly spaced flows.



Note the following:

- For these functions, you must enter the addresses of the cells that contain the input values.
- The IRR functions calculate only one IRR even when there are multiple IRRs.

Spreadsheet Questions

The following questions provide an opportunity to practice each of the above functions:

1. (IRR) Check the IRRs on projects F and G in Section 5-3.
2. (IRR) What is the IRR of a project with the following cash flows:

C_0	C_1	C_2	C_3
-\$5,000	+\$2,200	+\$4,650	+\$3,330

3. (IRR) Now use the function to calculate the IRR on Helmsley Iron's mining project in Section 5-3. There are really two IRRs to this project (why?). How many IRRs does the function calculate?
4. (XIRR) What is the IRR of a project with the following cash flows:

C_0	C_4	C_5	C_6
-\$215,000 . . .	+\$185,000	+\$85,000	+\$43,000

(All other cash flows are 0.)

rationed, then surely they *ought* not to use it. But that would be too quick a judgment. Let us look at this problem more deliberately.

Soft Rationing Many firms' capital constraints are "soft." They reflect no imperfections in capital markets. Instead they are provisional limits adopted by management as an aid to financial control.

Some ambitious divisional managers habitually overstate their investment opportunities. Rather than trying to distinguish which projects really are worthwhile, headquarters may find it simpler to impose an upper limit on divisional expenditures and thereby force the divisions to set their own priorities. In such instances budget limits are a rough but effective way of dealing with biased cash-flow forecasts. In other cases management may believe that very rapid corporate growth could impose intolerable strains on management and the organization. Since it is difficult to quantify such constraints explicitly, the budget limit may be used as a proxy.

Because such budget limits have nothing to do with any inefficiency in the capital market, there is no contradiction in using an LP model in the division to maximize net present value subject to the budget constraint. On the other hand, there is not much point in elaborate selection procedures if the cash-flow forecasts of the division are seriously biased.

Even if capital is not rationed, other resources may be. The availability of management time, skilled labor, or even other capital equipment often constitutes an important constraint on a company's growth.

Hard Rationing Soft rationing should never cost the firm anything. If capital constraints become tight enough to hurt—in the sense that projects with significant positive NPVs are passed up—then the firm raises more money and loosens the constraint. But what if it *can't* raise more money—what if it faces *hard* rationing?

Hard rationing implies market imperfections, but that does not necessarily mean we have to throw away net present value as a criterion for capital budgeting. It depends on the nature of the imperfection.

Arizona Aquaculture, Inc. (AAI), borrows as much as the banks will lend it, yet it still has good investment opportunities. This is not hard rationing so long as AAI can issue stock. But perhaps it can't. Perhaps the founder and majority shareholder vetoes the idea from fear of losing control of the firm. Perhaps a stock issue would bring costly red tape or legal complications.¹⁵

This does not invalidate the NPV rule. AAI's *shareholders* can borrow or lend, sell their shares, or buy more. They have free access to security markets. The type of portfolio they hold is independent of AAI's financing or investment decisions. The only way AAI can help its shareholders is to make them richer. Thus AAI should invest its available cash in the package of projects having the largest aggregate net present value.

A barrier between the firm and capital markets does not undermine net present value so long as the barrier is the *only* market imperfection. The important thing is that the firm's *shareholders* have free access to well-functioning capital markets.

The net present value rule *is* undermined when imperfections restrict shareholders' portfolio choice. Suppose that Nevada Aquaculture, Inc. (NAI), is solely owned by its founder, Alexander Turbot. Mr. Turbot has no cash or credit remaining, but he is convinced that expansion of his operation is a high-NPV investment. He has tried to sell stock but has found that prospective investors, skeptical of prospects for fish farming in the desert, offer him much less than he thinks his firm is worth. For Mr. Turbot capital markets hardly exist. It makes little sense for him to discount prospective cash flows at a market opportunity cost of capital.

¹⁵A majority owner who is "locked in" and has much personal wealth tied up in AAI may be effectively cut off from capital markets. The NPV rule may not make sense to such an owner, though it will to the other shareholders.

SUMMARY

If you are going to persuade your company to use the net present value rule, you must be prepared to explain why other rules may *not* lead to correct decisions. That is why we have examined three alternative investment criteria in this chapter.

Some firms look at the book rate of return on the project. In this case the company decides which cash payments are capital expenditures and picks the appropriate rate to depreciate these expenditures. It then calculates the ratio of book income to the book value of the investment. Few companies nowadays base their investment decision simply on the book rate of return, but shareholders pay attention to book measures of firm profitability and some managers therefore look with a jaundiced eye on projects that would damage the company's book rate of return.

Some companies use the payback method to make investment decisions. In other words, they accept only those projects that recover their initial investment within some specified period. Payback is an ad hoc rule. It ignores the timing of cash flows within the payback period, and it ignores subsequent cash flows entirely. It therefore takes no account of the opportunity cost of capital.

The internal rate of return (IRR) is defined as the rate of discount at which a project would have zero NPV. It is a handy measure and widely used in finance; you should therefore know how

to calculate it. The IRR rule states that companies should accept any investment offering an IRR in excess of the opportunity cost of capital. The IRR rule is, like net present value, a technique based on discounted cash flows. It will therefore give the correct answer if properly used. The problem is that it is easily misapplied. There are four things to look out for:

1. *Lending or borrowing?* If a project offers positive cash flows followed by negative flows, NPV can rise as the discount rate is increased. You should accept such projects if their IRR is less than the opportunity cost of capital.
2. *Multiple rates of return.* If there is more than one change in the sign of the cash flows, the project may have several IRRs or no IRR at all.
3. *Mutually exclusive projects.* The IRR rule may give the wrong ranking of mutually exclusive projects that differ in economic life or in scale of required investment. If you insist on using IRR to rank mutually exclusive projects, you must examine the IRR on each incremental investment.
4. *The cost of capital for near-term cash flows may be different from the cost for distant cash flows.* The IRR rule requires you to compare the project's IRR with the opportunity cost of capital. But sometimes there is an opportunity cost of capital for one-year cash flows, a different cost of capital for two-year cash flows, and so on. In these cases there is no simple yardstick for evaluating the IRR of a project.

In developing the NPV rule, we assumed that the company can maximize shareholder wealth by accepting every project that is worth more than it costs. But, if capital is strictly limited, then it may not be possible to take every project with a positive NPV. If capital is rationed in only one period, then the firm should follow a simple rule: Calculate each project's profitability index, which is the project's net present value per dollar of investment. Then pick the projects with the highest profitability indexes until you run out of capital. Unfortunately, this procedure fails when capital is rationed in more than one period or when there are other constraints on project choice. The only general solution is linear programming.

Hard capital rationing always reflects a market imperfection—a barrier between the firm and capital markets. If that barrier also implies that the firm's shareholders lack free access to a well-functioning capital market, the very foundations of net present value crumble. Fortunately, hard rationing is rare for corporations in the United States. Many firms do use soft capital rationing, however. That is, they set up self-imposed limits as a means of financial planning and control.

For a survey of capital budgeting procedures, see:

- J. Graham and C. Harvey, "How CFOs Make Capital Budgeting and Capital Structure Decisions," *Journal of Applied Corporate Finance* 15 (spring 2002), pp. 8–23.



Select problems are available in McGraw-Hill's *Connect Finance*. Please see the preface for more information.

BASIC

1. Payback

- a. What is the payback period on each of the following projects?

Project	Cash Flows, \$				
	C_0	C_1	C_2	C_3	C_4
A	−5,000	+1,000	+1,000	+3,000	0
B	−1,000	0	+1,000	+2,000	+3,000
C	−5,000	+1,000	+1,000	+3,000	+5,000

FURTHER READING

PROBLEM SETS