IEE 1711: Applied Signal Processing

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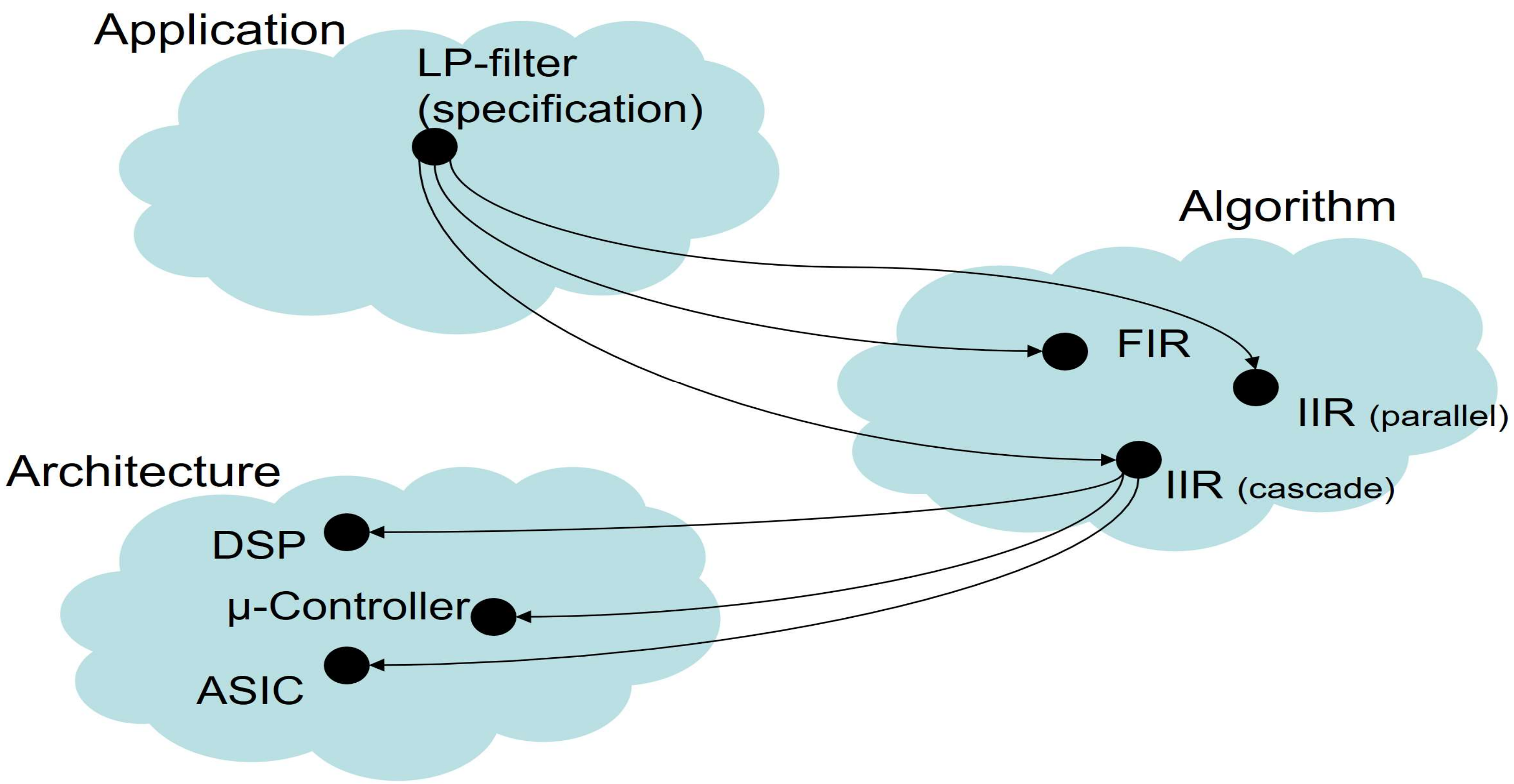


Outline

- Lecture 5: Digital Communication - Transmitter
  - Followup

- Lecture 8: Digital Communication – Receiver
  - Maximum Likelihood Sequence Estimator
  - Zero-forcing Equalization

    Linear filters

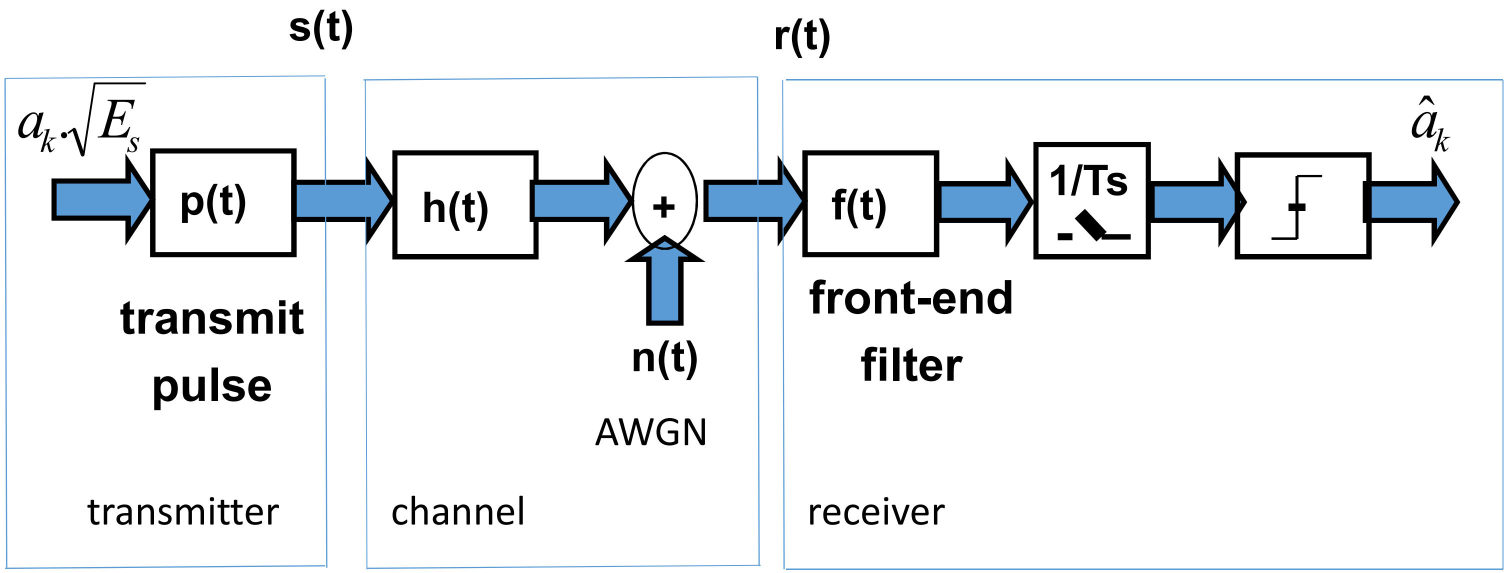
    Decision feedback equalizers
  - MMSE Equalization

  - Summary

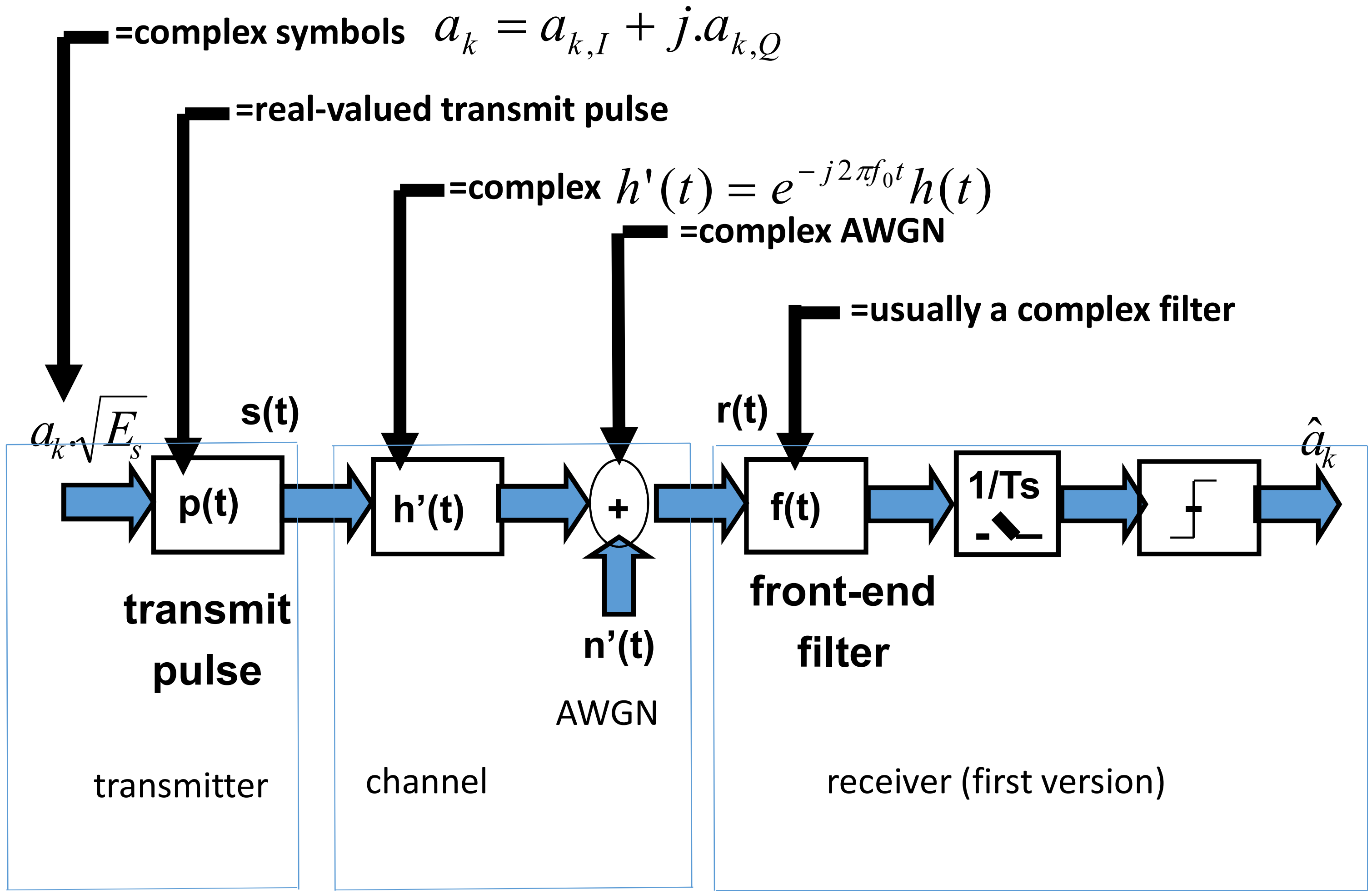
Followup - Transmitter

Passband transmission model/definitions

a convenient and consistent (baseband) model can be obtained, based on complex envelope signals, that does not have the modulation/demodulation steps:



Followup - Transmitter



BER Performance for AWGN Channel

definitions:

- transmitted signal $$s(t) = \sqrt{E_s}.\sum_k a_k.p(t - kT_s)$$

- received signal (at front-end filter) $$r(t) = \sqrt{E_s}.\sum_k a_k.p(t - kT_s) + n(t)$$

- received signal (at sampler) $$r'(t) = \sqrt{E_s}.\sum_k a_k.g(t - kT_s) + n'(t)$$

g(t) =p(t)*f(t) = transmitted pulse p(t) filtered by front-end filter

n'(t) =n(t)*f(t) = AWGN filtered by front-end filter

BER Performance for AWGN Channel

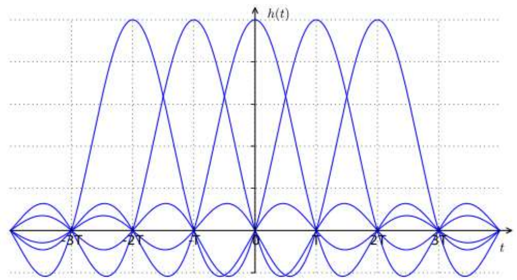
Received signal sampled @ time t=k.Ts is...

$$r'(k.T_s) = \underbrace{\sqrt{E_s}.a_k.g(0)}_{1} + \underbrace{\sum_{m\neq 0} a_{k-m}.g(m.T_s)}_{2} + \underbrace{n'(k.T_s)}_{3}$$

1 = useful term

2= `ISI', intersymbol interference (from symbols other than a_k)

3= noise term



Consecutive raised-cosine impulses, demonstrating zero-ISI property

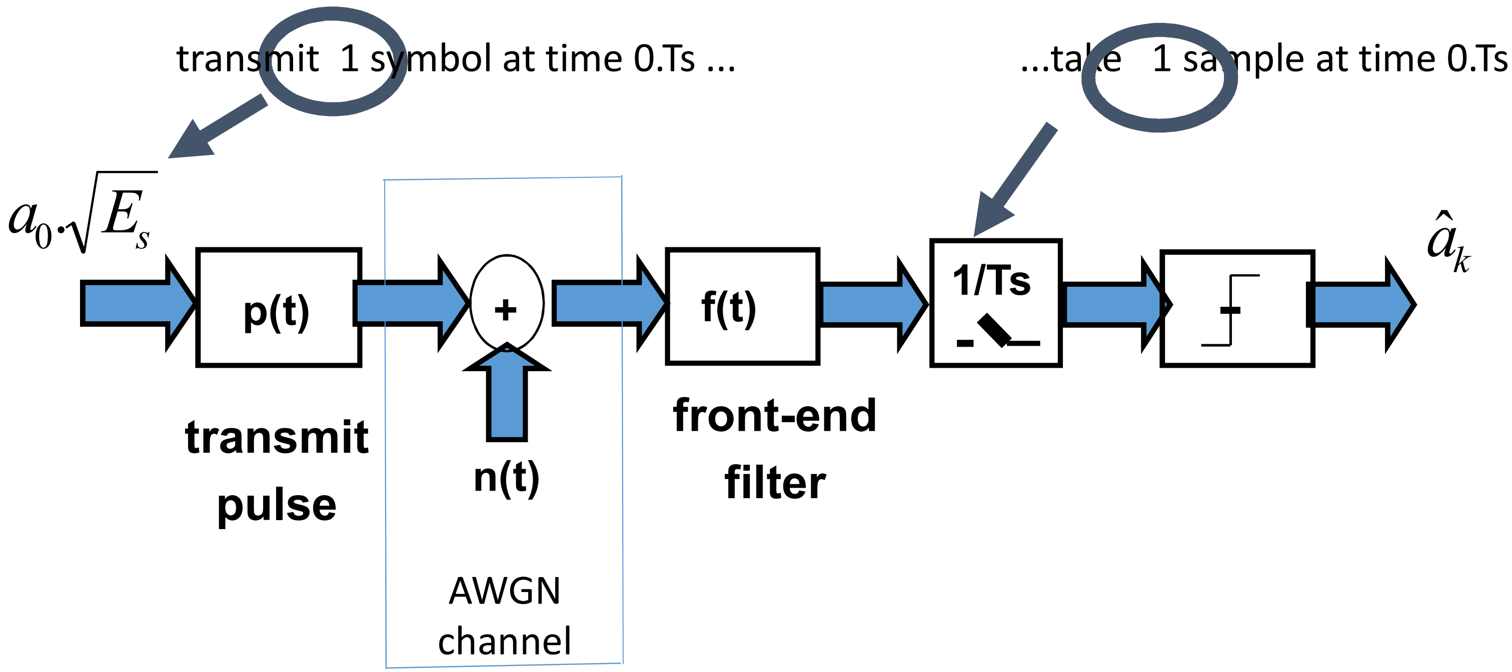
Strategy:

a) analyze BER in absence of ISI (=`transmission of 1 symbol')

b) analyze pulses for which ISI-term = 0 (such that analysis under a. applies)

c) for non-zero ISI,

Transmission of 1 symbol over AWGN channel (I)



Transmission of 1 symbol over AWGN channel (II)

Received signal sampled @ time t=0.Ts is..

$$r'(0.T_s) = \underbrace{\sqrt{E_s}.a_0.g(0)}_{1} + \underbrace{0}_{2} + \underbrace{n'(0.T_s)}_{3}$$

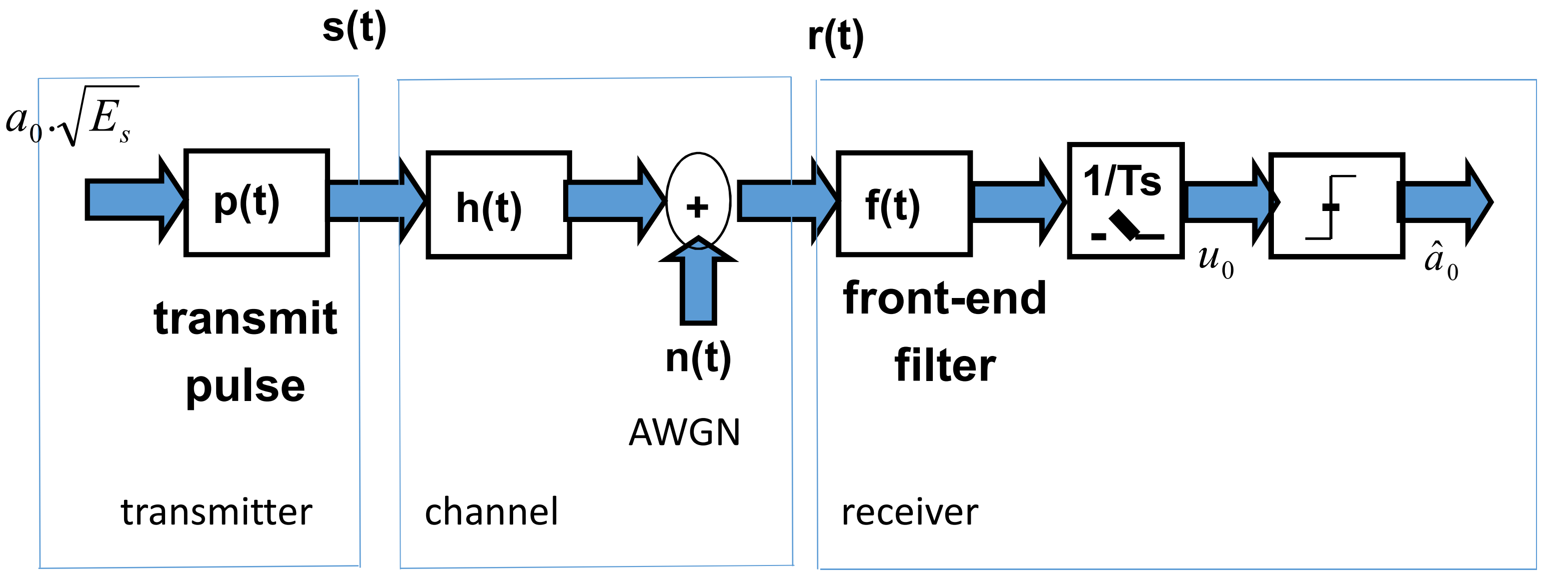
- `Minimum distance' decision rule/device :

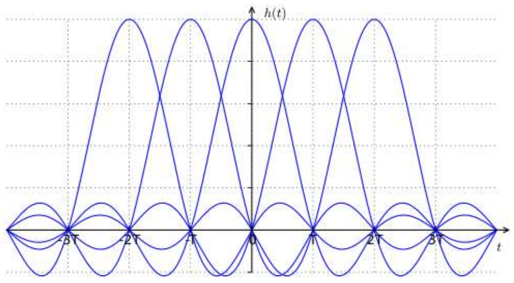
$$\hat{a}_0 = \alpha_i \Leftrightarrow \left| \frac{r'(0.T_s)}{\sqrt{E_s}.g(0)} - \alpha_i \right| = \min_{0 \le n \le M-1} \left| \frac{r'(0.T_s)}{\sqrt{E_s}.g(0)} - \alpha_n \right|$$

Optimal receiver design

Receiver:

A receiver structure is postulated (front-end filter + symbol-rate sampler + memory-less decision device). For transmission of 1 symbol, it was found that the front-end filter should be `matched' to the received pulse.





Consecutive raised-cosine impulses, demonstrating zero-ISI property

Optimal receiver

Problem Statement :

- Optimal receiver structure consists of

  * Whitened Matched Filter (WMF) front-end

    (= matched filter + symbol-rate sampler + equalizer filter)

  * Maximum Likelihood Sequence Estimator (MLSE),

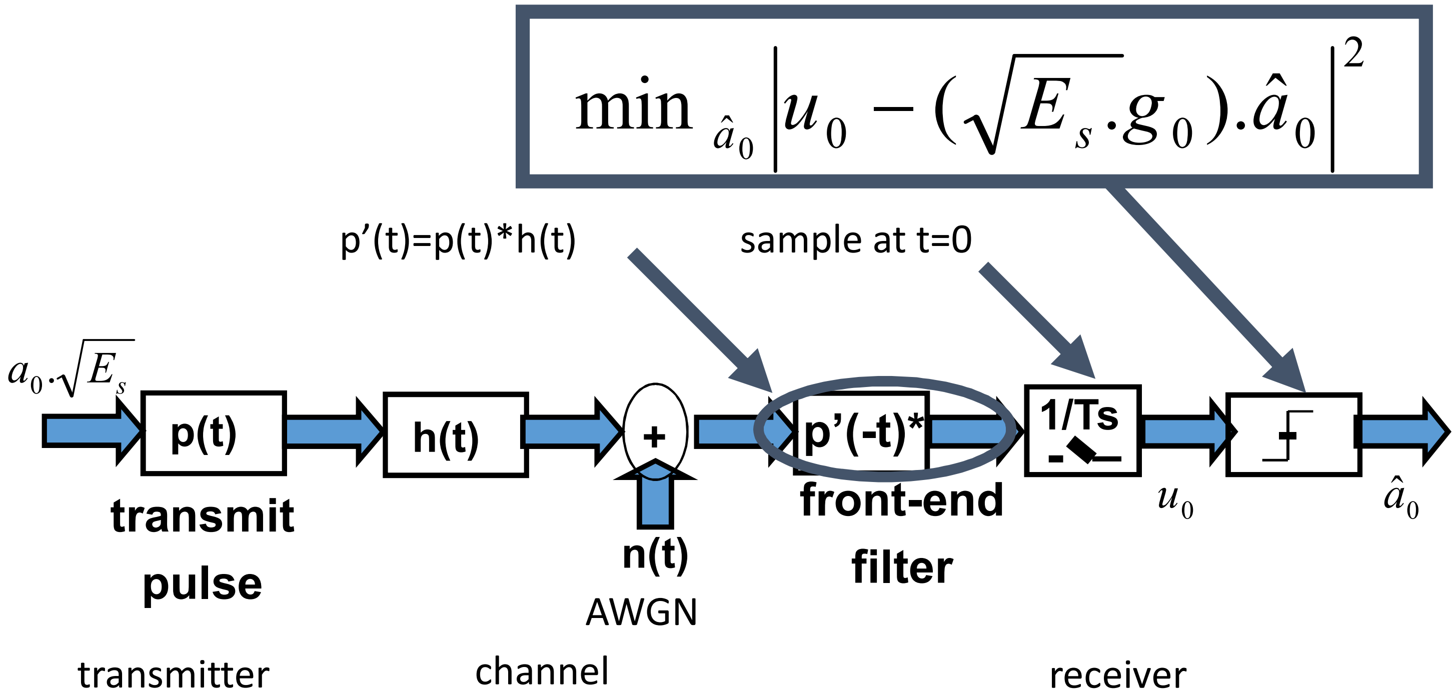
    (instead of simple memory-less decision device)

Lecture-8: Optimal Receiver

- Equalization – Overview
- Maximum Likelihood Sequence Estimator
- Zero-forcing Equalization
  - Linear filters
  - Decision feedback equalizers
- MMSE Equalization

Maximum Likelihood Sequence Estimator (1)

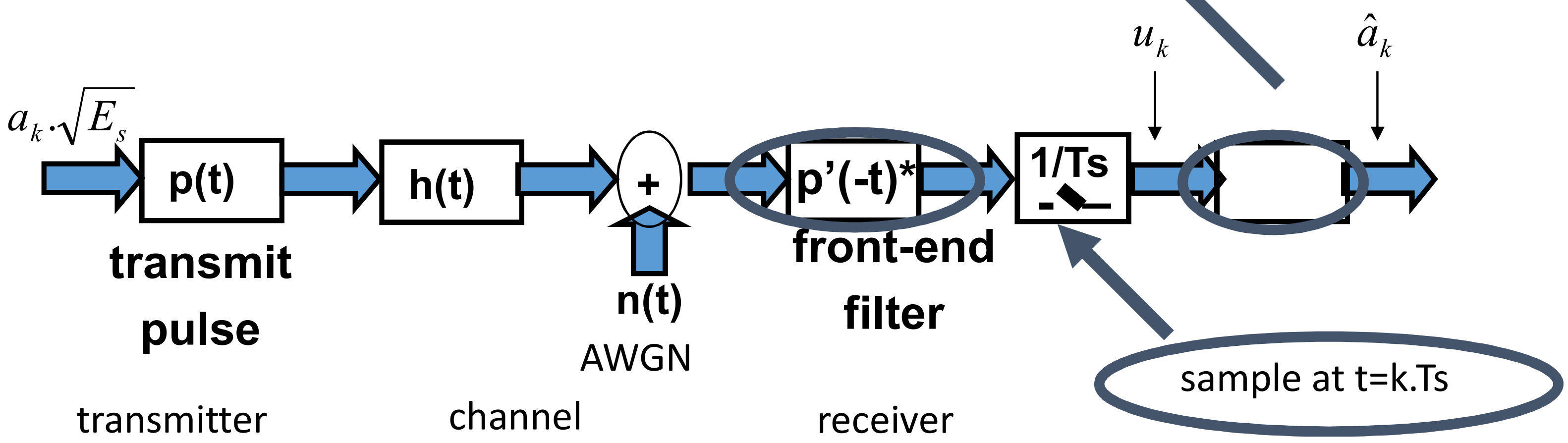
Receiver: In Lecture-5, it was found that for transmission of 1 symbol, the receiver structure below is indeed optimal !



Maximum Likelihood Sequence Estimator (2)

- Receiver: For transmission of a symbol sequence, the optimal receiver structure is...

$$\min_{\hat{a}_0,\ldots,\hat{a}_K} \left[\sqrt{E_s}.\sum_{k=1}^{K}\sum_{l=1}^{K} \hat{a}_k^*.g_{k-l}.\hat{a}_l - \sum_{k=1}^{K} 2\Re\left\{\hat{a}_k^*.u_k\right\} \right]$$



Maximum Likelihood Sequence Estimator (3)

Receiver:

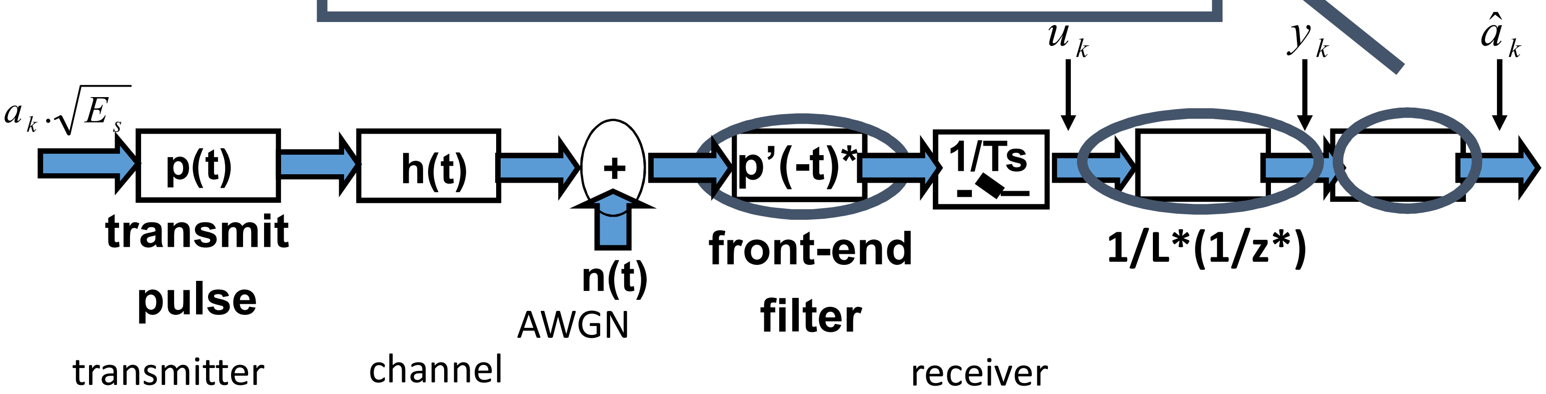
- This receiver structure is based on symbol-rate sampling (=usually below Nyquist-rate sampling), which appears to be allowable if preceded by a matched-filter front-end.
- Criterion for decision device is too complicated. Need a simpler criterion/procedure...

Maximum Likelihood Sequence Estimator (4)

Receiver: 1st simplification by insertion of an additional filter (after sampler).

* Filter = `pre-equalizer'
* Complete front-end = `Whitened matched filter'

$$\min_{\hat{a}_0,\ldots,\hat{a}_K} \sum_{m=1}^{K} \left| y_m - \sum_{k=1}^{K} \hat{a}_k . h_{m-k} \right|^2$$

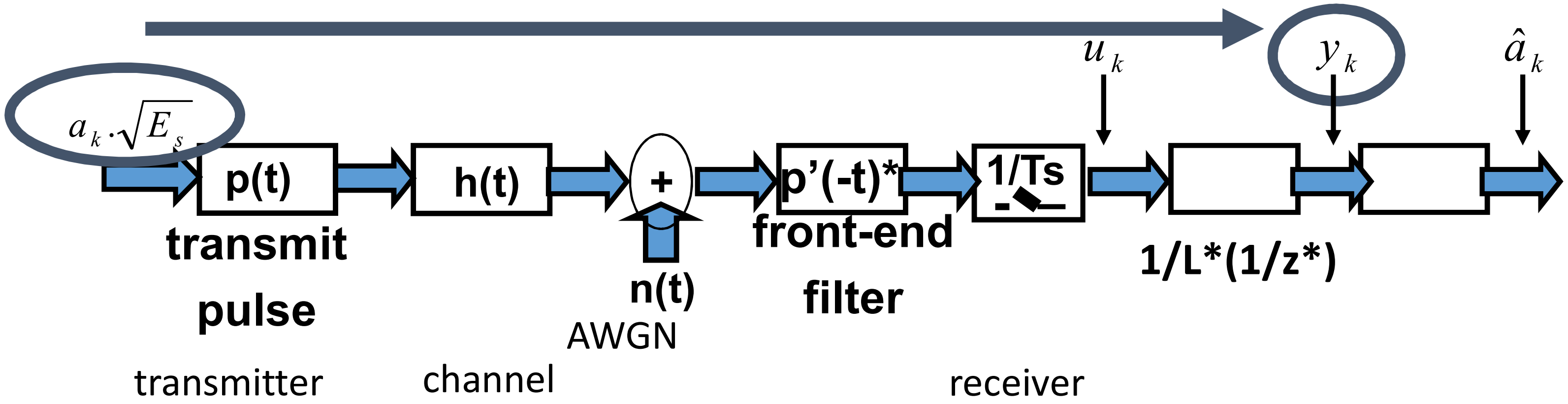


Maximum Likelihood Sequence Estimator (5)

Receiver: The additional filter turned the complete transmitter-receiver chain into a simple input-output model:

$$y_k = h_0.a_k + h_1..a_{k-1} + h_2..a_{k-2} + h_3.a_{k-3} + \ldots + w_k$$

$$y_k = \underbrace{(h_0 + h_1.z^{-1} + h_2.z^{-2} + h_3.z^{-3} + \ldots)}_{H(z)}.a_k + w_k$$



Maximum Likelihood Sequence Estimator (6)

Receiver: simple input-output model:

$$y_k = h_0 . a_k + h_1 . a_{k-1} + h_2 . a_{k-2} + h_3 . a_{k-3} + \ldots + w_k$$

w_k = additive white Gaussian noise

$h_{-1} = h_{-2} = \ldots = 0$

means interference from future symbols has been cancelled, hence only interference from past symbols remains

Maximum Likelihood Sequence Estimator (7)

Receiver: Based on the input-output model

$$y_k = h_0.a_k + h_1..a_{k-1} + h_2..a_{k-2} + h_3.a_{k-3} + \ldots + w_k$$

one can compute the transmitted symbol sequence as

$$\min_{\hat{a}_0,\ldots,\hat{a}_K} \sum_{m=1}^{K} \left| y_m - \sum_{k=1}^{K} \hat{a}_k.h_{m-k} \right|^2$$

A recursive procedure for this = Viterbi Algorithm

Problem = complexity proportional to M^N !

(N=channel-length=number of non-zero filter taps)

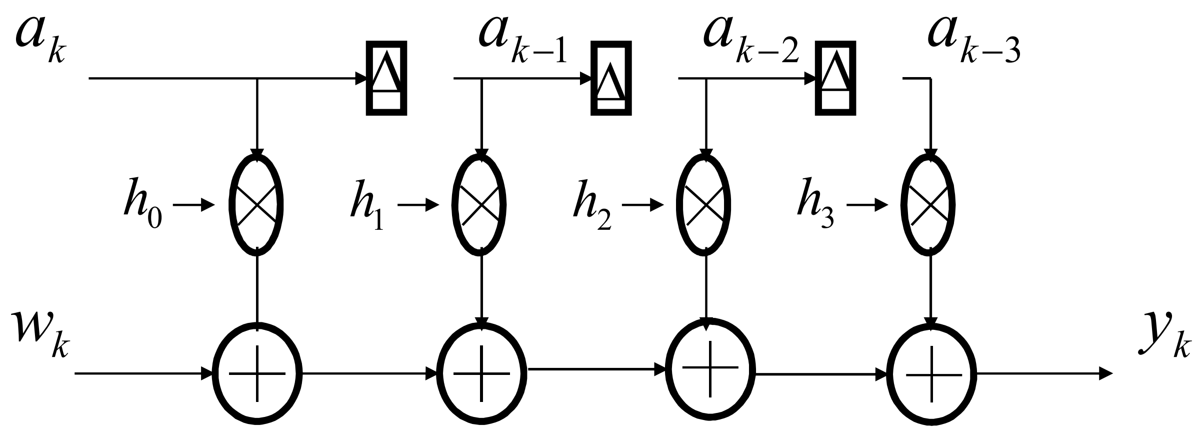
Problem statement (revisited)

- Cheap alternative for MLSE/Viterbi ?
- Solution: equalization filter + memory-less decision device (`slicer’)
  - Linear filters
  - Non-linear filters (decision feedback)
- Complexity : linear in number filter taps
- Performance : with channel coding, approaches MLSE performance

Preliminaries (I)

- **Our starting point** will be the input-output model for transmitter + channel + receiver whitened matched filter front-end

$$y_k = h_0 . a_k + h_1 . a_{k-1} + h_2 . a_{k-2} + h_3 . a_{k-3} + \ldots + w_k$$



Preliminaries (II)

- z-transform for discrete-time signals…

$$A(z) = \sum_{i=0}^{\infty} a_i . z^{-i} = a_0 . z^{-0} + a_1 . z^{-1} + a_2 . z^{-2} + ...$$

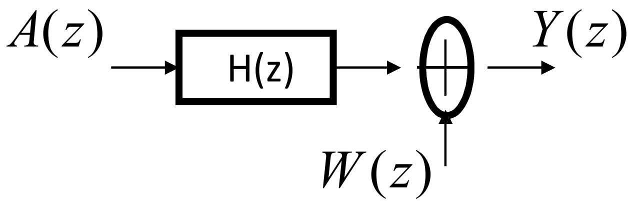
$$H(z) = \sum_{i=0}^{\infty} h_i . z^{-i} = h_0 . z^{-0} + h_1 . z^{-1} + h_2 . z^{-2} + ...$$

…and for input/output behavior of discrete-time systems

$$y_k = h_0 . a_k + h_1 . a_{k-1} + h_2 . a_{k-2} + h_3 . a_{k-3} + ... + w_k$$

hence

$$Y(z) = H(z) . A(z) + W(z)$$



Preliminaries (III)

properties/advantages of the WMF front end

- additive noise w_k = white (colored in general model)
- H(z) does not have anti-causal taps $h_{-1} = h_{-2} = ... = 0$
  - anti-causal taps originate, e.g., from transmit filter design (RRC, etc.).
  - practical implementation based on causal filters + delays...

Preliminaries (IV)

$$y_k = h_0.a_k + \underbrace{h_1.a_{k-1} + h_2.a_{k-2} + h_3.a_{k-3} + \ldots}_{\text{ISI}} + \underbrace{w_k}_{\text{NOISE}}$$

- `Equalization’: compensate for channel distortion.

  Resulting signal fed into memory-less decision device.

- Let us consider (ideal-case):

  - channel distortion model assumed to be known

  - no constraints on the complexity of the equalization filter (number of filter taps)

Zero-forcing & MMSE Equalizers

$$y_k = h_0.a_k + \underbrace{h_1.a_{k-1} + h_2.a_{k-2} + h_3.a_{k-3} + \ldots}_{\text{ISI}} + \underbrace{w_k}_{\text{NOISE}}$$

Classes of Equalizers:

1- Zero-forcing (ZF) equalizers

eliminate inter-symbol-interference (ISI) at the slicer input

2- Minimum mean-square error (MMSE) equalizers

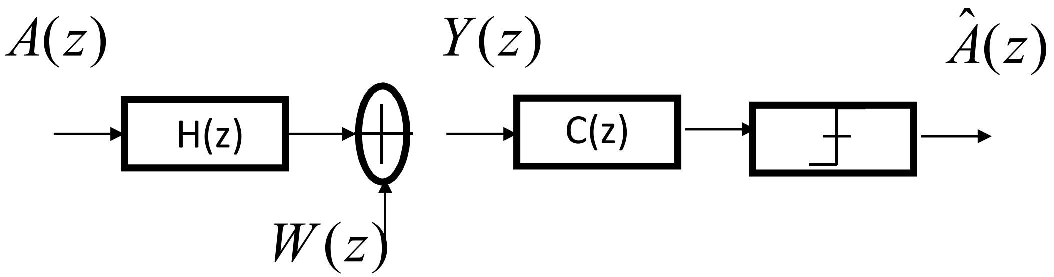
tradeoff between minimizing ISI and minimizing noise at the slicer input

Zero-forcing Equalizers

Zero-forcing Linear Equalizer (LE) :

- equalization filter is inverse of H(z)
- decision device (`slicer')

$$C(z) = H^{-1}(z)$$



- Problem : noise enhancement (C(z).W(z) large)

Zero-forcing Equalizers

Zero-forcing Linear Equalizer (LE) :

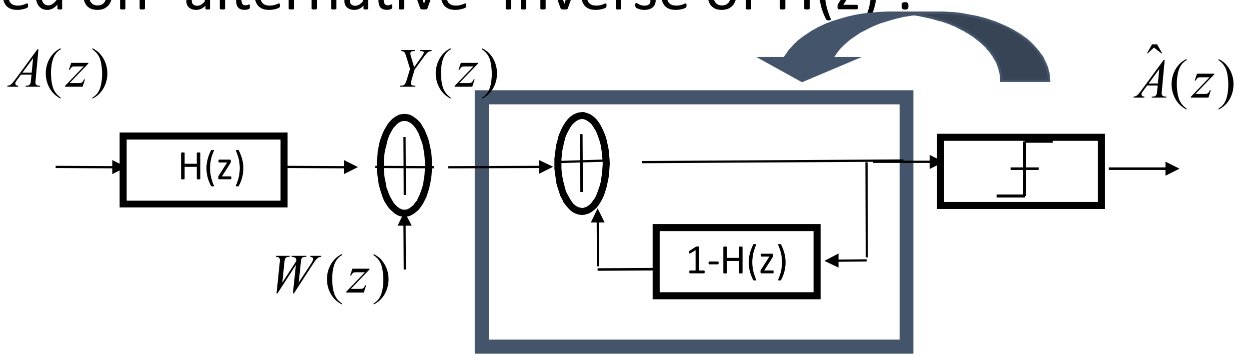
- Observation 1: under the constraint of zero-ISI at the slicer input, the LE with whitened matched filter front-end is optimal in that it minimizes the noise at the slicer input
- Observation 2: : if a different front-end is used, H(z) may have unstable zeros (non-minimum-phase), hence may be `difficult' to invert.

Zero-forcing Equalizers

Zero-forcing Non-linear Equalizer

Decision Feedback Equalization (DFE) :

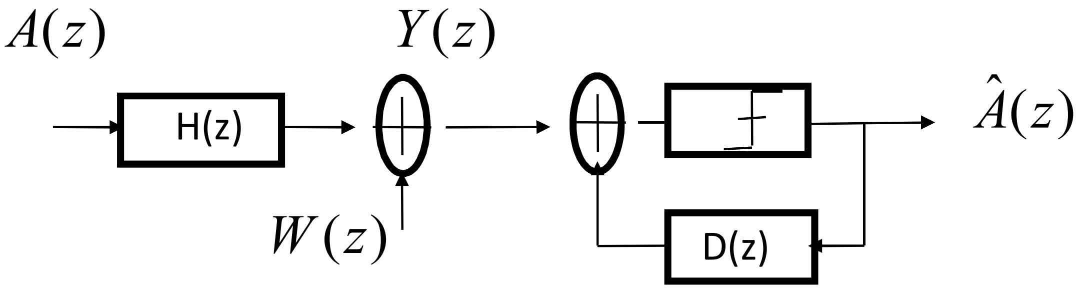
- derivation based on `alternative' inverse of H(z) :



(ps: this is possible if H(z) has $h_0 = 1$, which is another property of the WMF model)

- now move slicer inside the feedback loop :

Zero-forcing Equalizers



$$D(z) = 1 - H(z)$$

moving slicer inside the feedback loop has…

- beneficial effect on noise: noise is removed that would otherwise circulate back through the loop
- beneficial effect on stability of the feedback loop: output of the slicer is always bounded, hence feedback loop always stable

Performance intermediate between MLSE and linear equaliz.