IEE 1711: Applied Signal Processing

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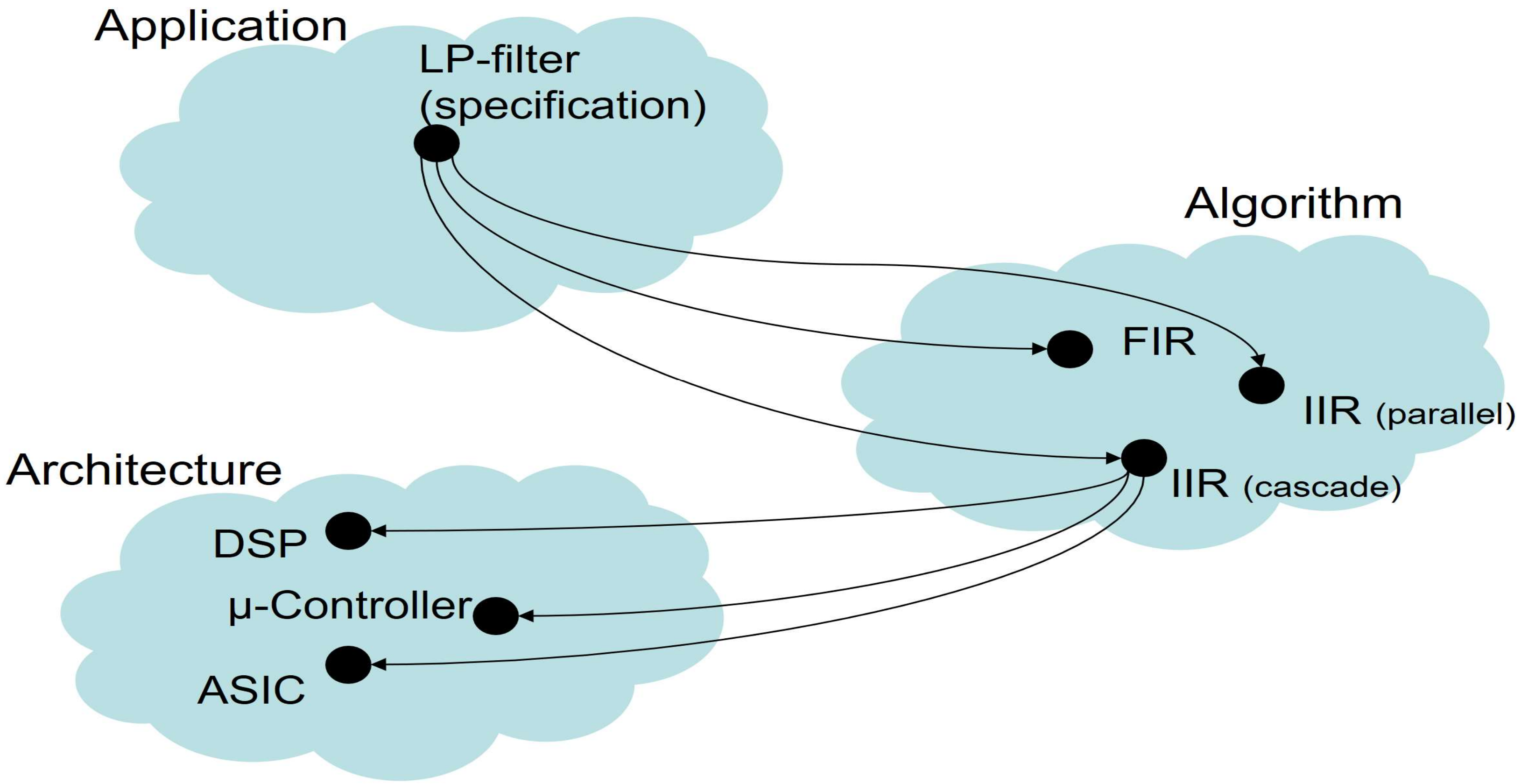


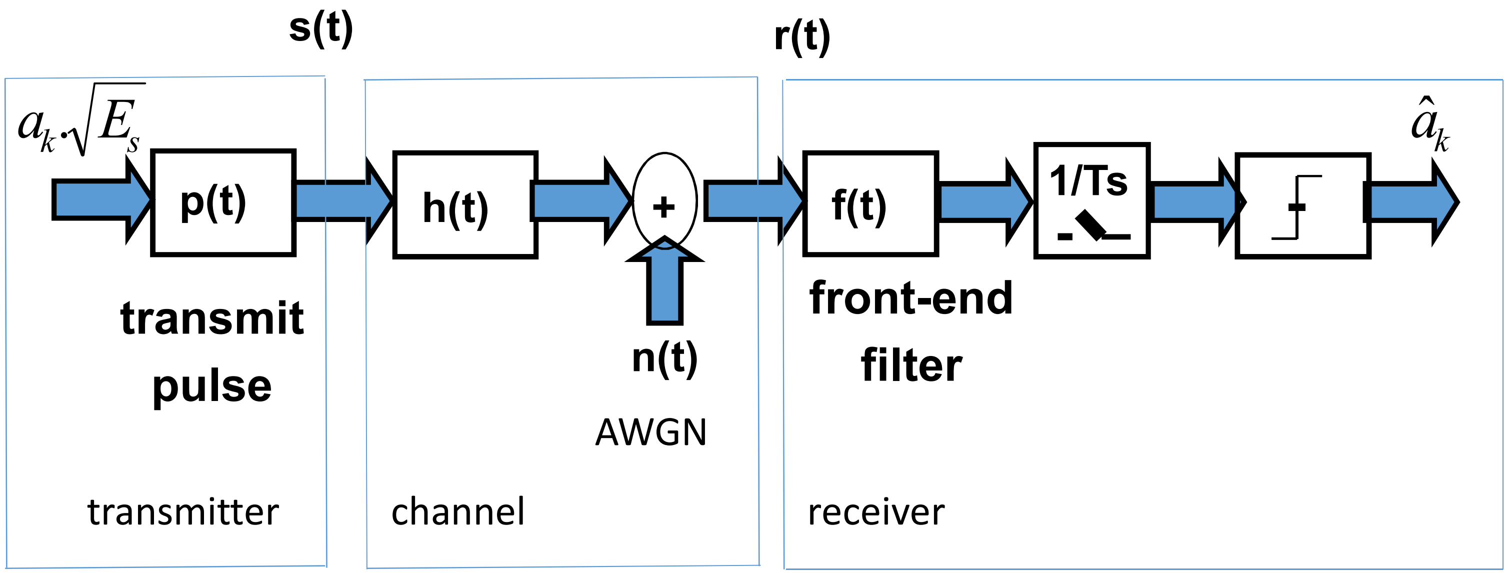
Outline

- Lecture 4: Digital Communication - Transmitter
  - Followup
- Lecture 5: Digital Communication – Transmitter Continued
  - Transmitter
    - BER Performacne for AWGN Channel
      - Transmisison over AWGN Channel
    - Symbol sequence over AWGN channel
    - Zero ISIS forcing Pulse Design
    - Nyquist Filter Design
- Summary

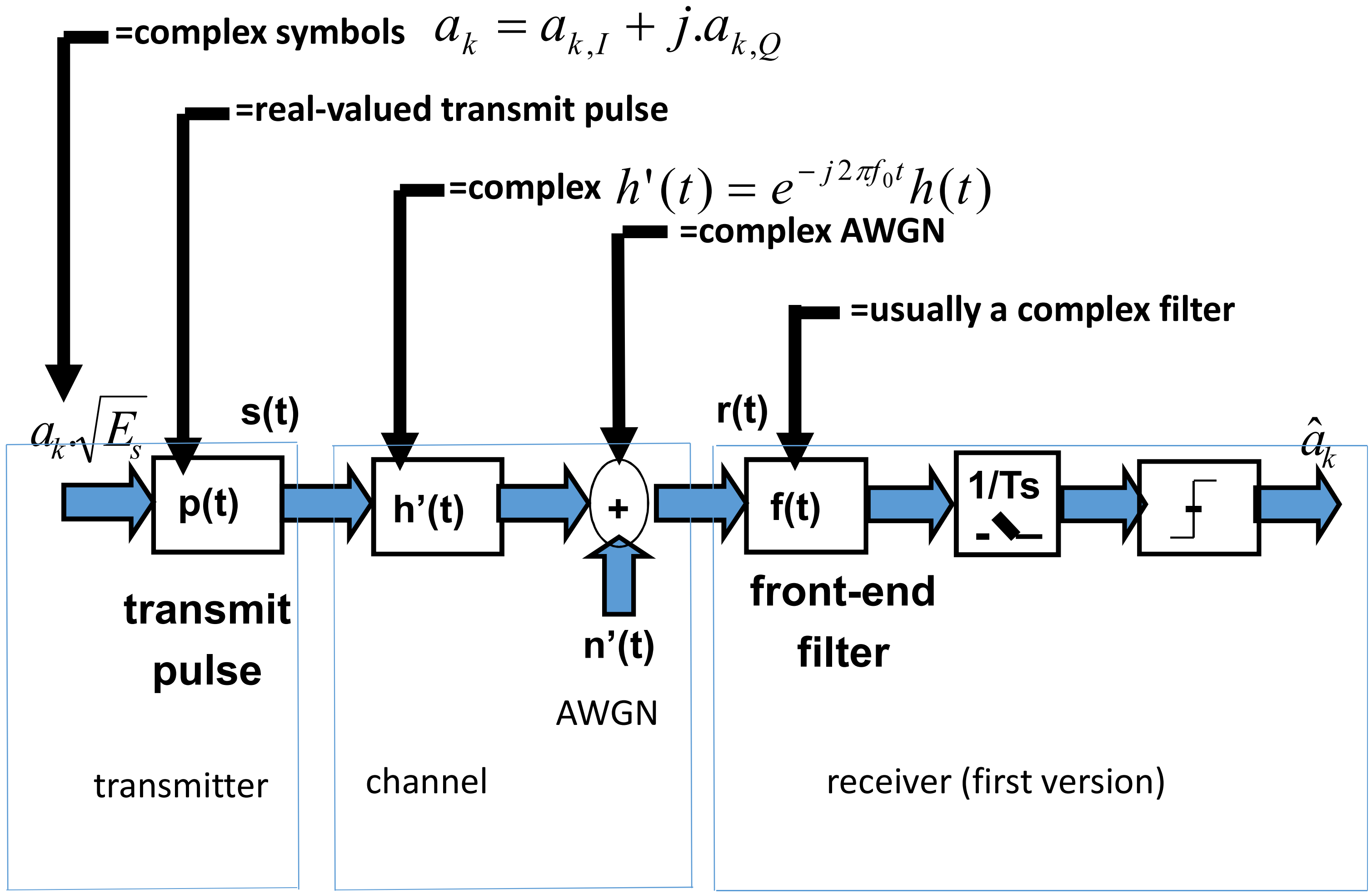
Preliminaries : Passband vs. baseband transmission

Passband transmission model/definitions

a convenient and consistent (baseband) model can be obtained, based on complex envelope signals, that does not have the modulation/demodulation steps:

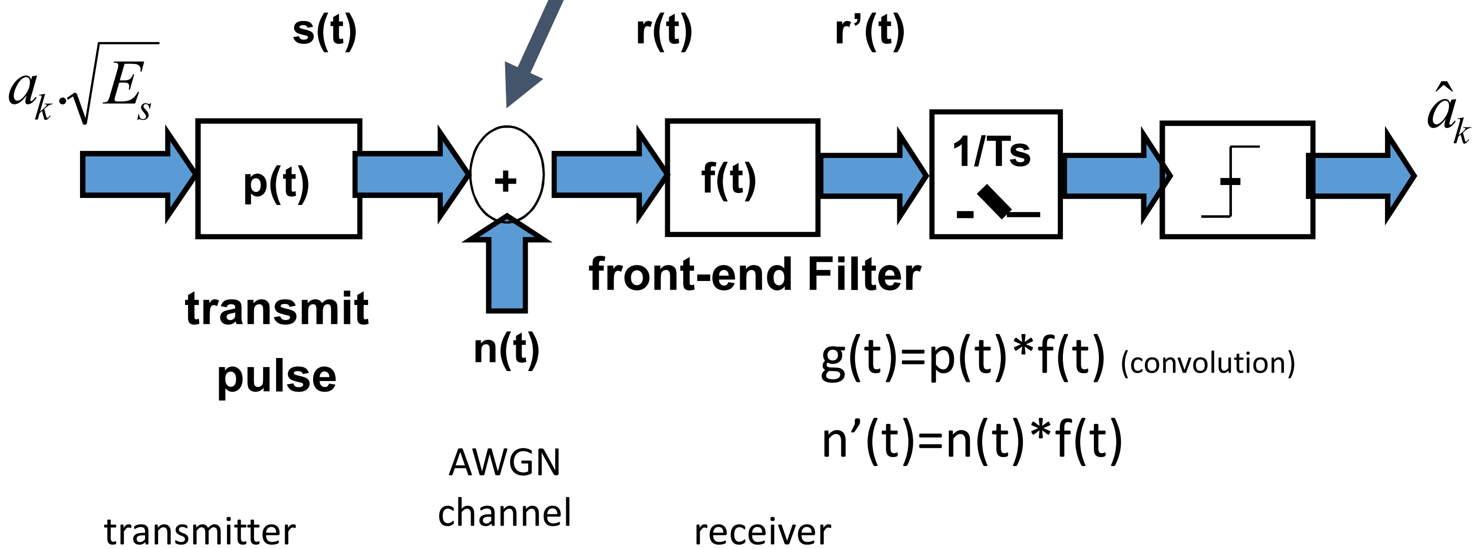


Preliminaries : Passband vs. baseband transmission



BER Performance for AWGN Channel

BER=(# bit errors)/(# transmitted bits)



g(t)=p(t)*f(t) (convolution)

n'(t)=n(t)*f(t)

BER for different constellations?

BER Performance for AWGN Channel

definitions:

- transmitted signal $$s(t) = \sqrt{E_s}.\sum_k a_k.p(t - kT_s)$$

- received signal (at front-end filter) $$r(t) = \sqrt{E_s}.\sum_k a_k.p(t - kT_s) + n(t)$$

- received signal (at sampler) $$r'(t) = \sqrt{E_s}.\sum_k a_k.g(t - kT_s) + n'(t)$$

g(t) =p(t)*f(t) = transmitted pulse p(t) filtered by front-end filter

n'(t) =n(t)*f(t) = AWGN filtered by front-end filter

BER Performance for AWGN Channel

Received signal sampled @ time t=k.Ts is...

$$r'(k.T_s) = \underbrace{\sqrt{E_s}.a_k.g(0)}_{1} + \underbrace{\sum_{m\neq 0} a_{k-m}.g(m.T_s)}_{2} + \underbrace{n'(k.T_s)}_{3}$$

1 = useful term

2= \`ISI’, intersymbol interference (from symbols other than a_k)

3= noise term

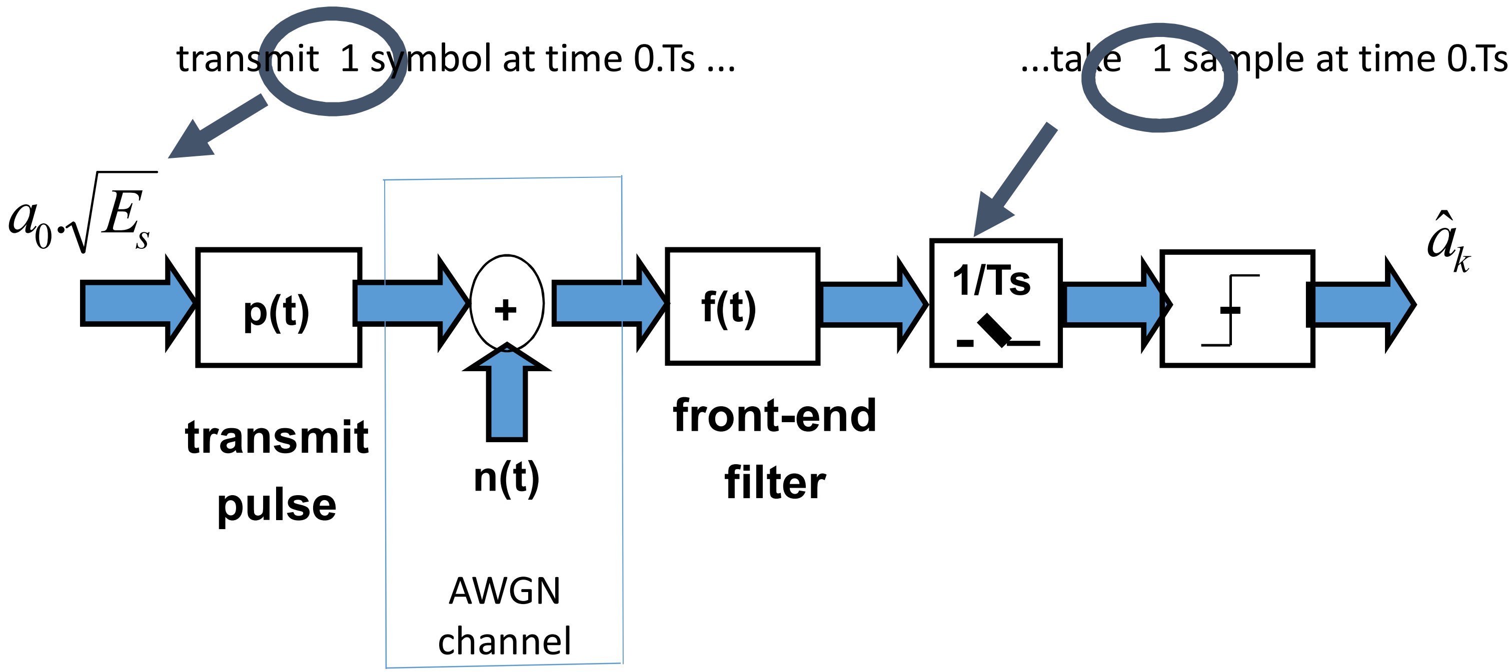
Strategy :

a) analyze BER in absence of ISI (=\`transmission of 1 symbol’)

b) analyze pulses for which ISI-term = 0 (such that analysis under a. applies)

c) for non-zero ISI,

Transmission of 1 symbol over AWGN channel (I)



BER for different constellations?

Transmission of 1 symbol over AWGN channel (II)

Received signal sampled @ time t=0.Ts is..

$$r'(0.T_s) = \underbrace{\sqrt{E_s}.a_0.g(0)}_{1} + \underbrace{0}_{2} + \underbrace{n'(0.T_s)}_{3}$$

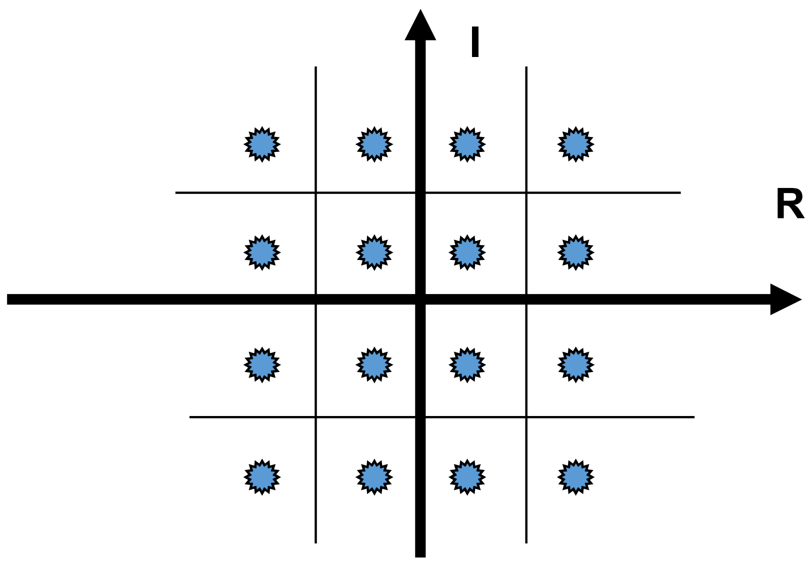
- `Minimum distance' decision rule/device :

$$\hat{a}_0 = \alpha_i \Leftrightarrow \left| \frac{r'(0.T_s)}{\sqrt{E_s}.g(0)} - \alpha_i \right| = \min_{0 \le n \le M-1} \left| \frac{r'(0.T_s)}{\sqrt{E_s}.g(0)} - \alpha_n \right|$$

Transmission of 1 symbol over AWGN channel (III)

`Minimum distance’ decision rule :

Example : decision regions for 16-QAM



Transmission of 1 symbol over AWGN channel (IV)

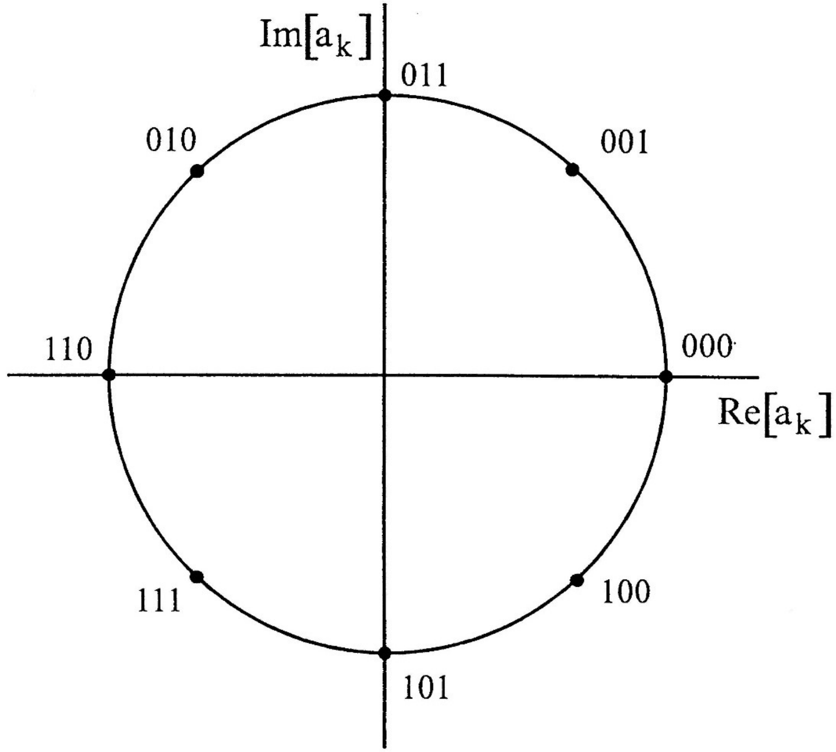
Preliminaries :BER versus SER (symbol-error-rate)

- **aim:** each *symbol error* (1 symbol = n bits) introduces only 1 *bit error*
- **how?** : GRAY CODING

  make nearest neighbor symbols correspond to groups of n bits that differ only in 1 bit position…
- …hence `nearest neighbor symbol errors' (=most symbol errors) correspond to 1 bit error

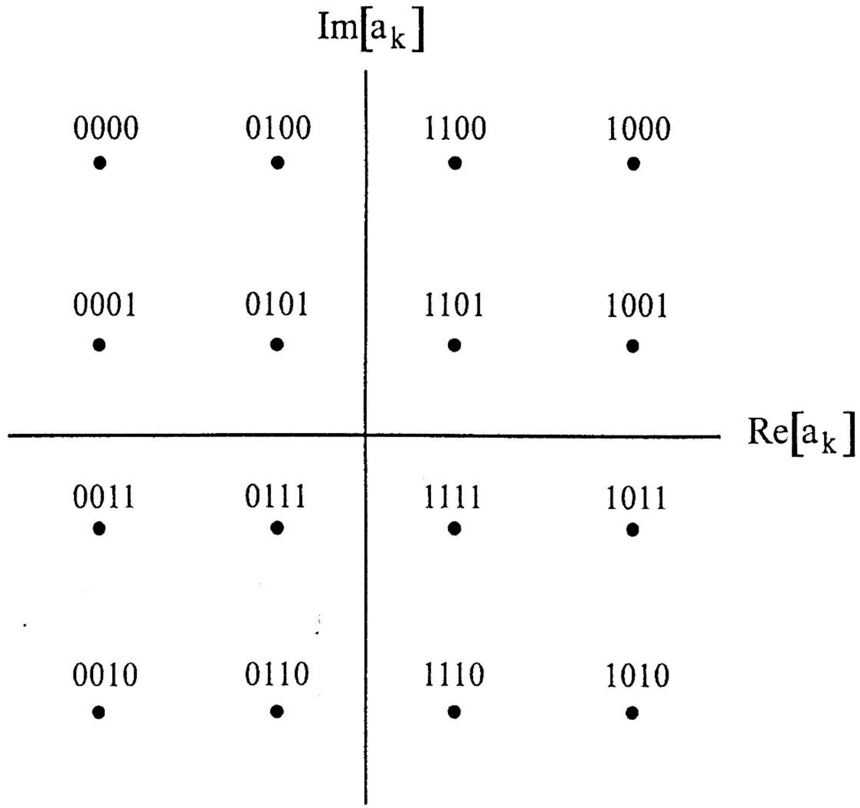
Transmission of 1 symbol over AWGN channel (V)

Gray Coding for 8-PSK



Transmission of 1 symbol over AWGN channel (VI)

Gray Coding for 16-QAM



Transmission of 1 symbol over AWGN channel (VII)

- Computations : skipped

  (compute probability that additive noise pushes received sample in wrong decision region)

- Results:

$$BER = \frac{N(M)}{\log_2 M}.Q(\sqrt{\gamma.\frac{E_b}{2N_0}.d^2(M).\log_2(M)}$$

$$\gamma = \frac{|g(0)|^2}{\int_{-\infty}^{+\infty} |F(f)|^2 df}$$

$$Q(x) = \frac{1}{\sqrt{2.\pi}} \int_x^{+\infty} \exp(-\frac{u^2}{2}) du$$

$$N(M) = \text{average number of neighbors}$$

Transmission of 1 symbol over AWGN channel (VIII)

Interpretation (I) : Eb/No

- Eb= energy-per-bit=Es/n=(signal power)/(bitrate)
- No=noise power per Hz bandwidth

lower BER for higher Eb/No

Transmission of 1 symbol over AWGN channel (IX)

Interpretation (II) : Constellation

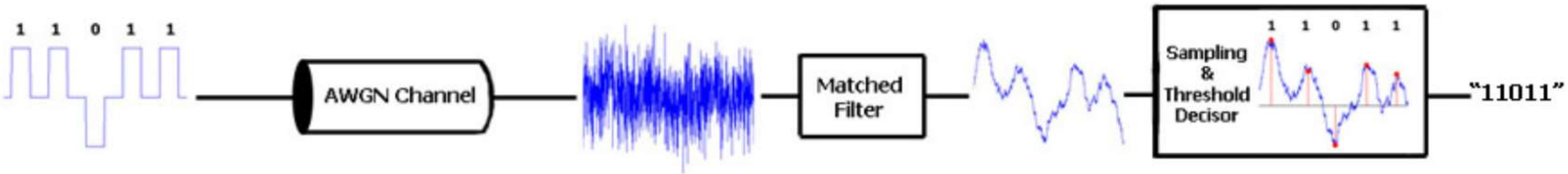
for given Eb/No, it is found that...

BER(M-QAM) =< BER(M-PSK) =< BER(M-PAM)

higher BER for larger M (in each constellation family)

Matched filters

- In signal processing, a **matched filter** is obtained by correlating a known signal, or *template*, with an unknown signal to detect the presence of the template in the unknown signal.
- This is equivalent to convolving the unknown signal with a conjugated time-reversed version of the template.
- The **matched filter** is the optimal linear **filter** for maximizing the signal to noise ratio (SNR) in the presence of additive stochastic noise.
- **Matched filters** are commonly used in radar, in which a signal is sent out, and we measure the reflected signals, looking for something similar to what was sent out.



Transmission of 1 symbol over AWGN channel (X)

Interpretation (III): front-end filter f(t)

$$\gamma = \frac{\left|g(0)\right|^2}{\int_{-\infty}^{+\infty} \left|F(f)\right|^2 df}$$

It is proven that $0 \le \gamma \le 1$

and that $\gamma = 1$ is obtained only when

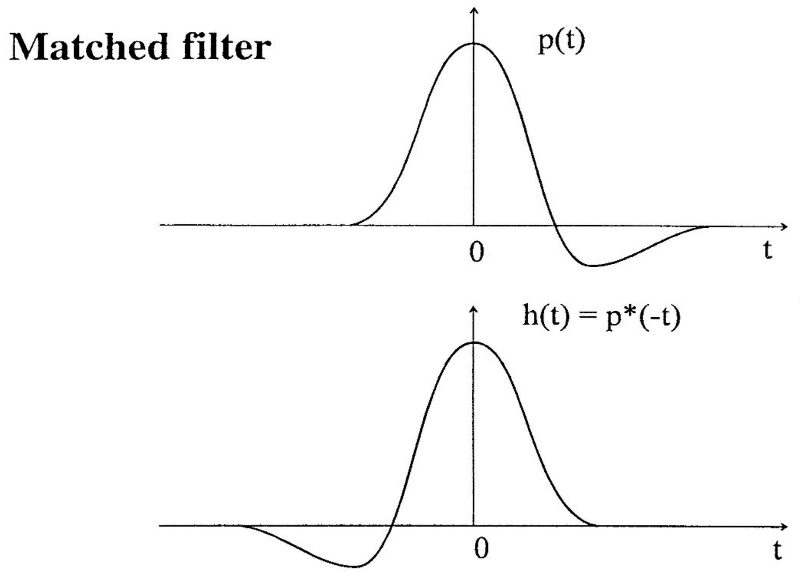
$$F(f) = P^*(f), \text{ i.e. } \boxed{f(t) = p^*(-t)} \text{ and } G(f) = \left|P(f)\right|^2$$

this is known as the **`matched filter receiver'**

Transmission of 1 symbol over AWGN channel (XI)

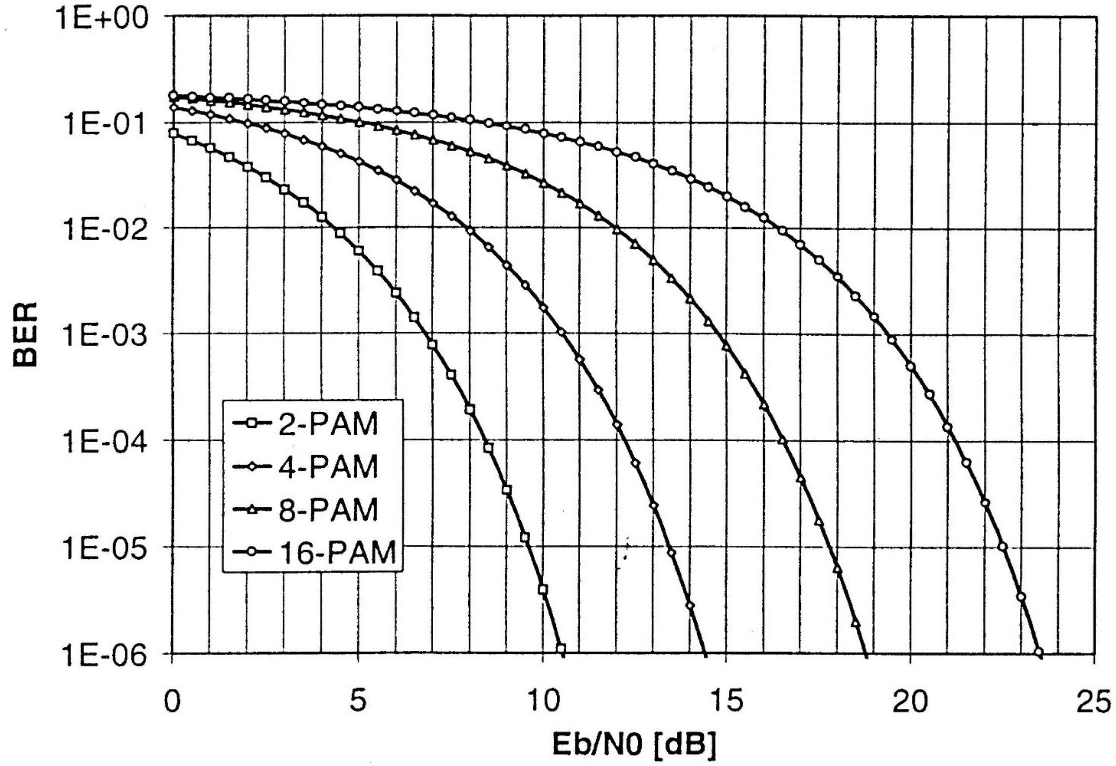
Interpretation (IV)

with a matched filter receiver, obtained BER is independent of pulse p(t)



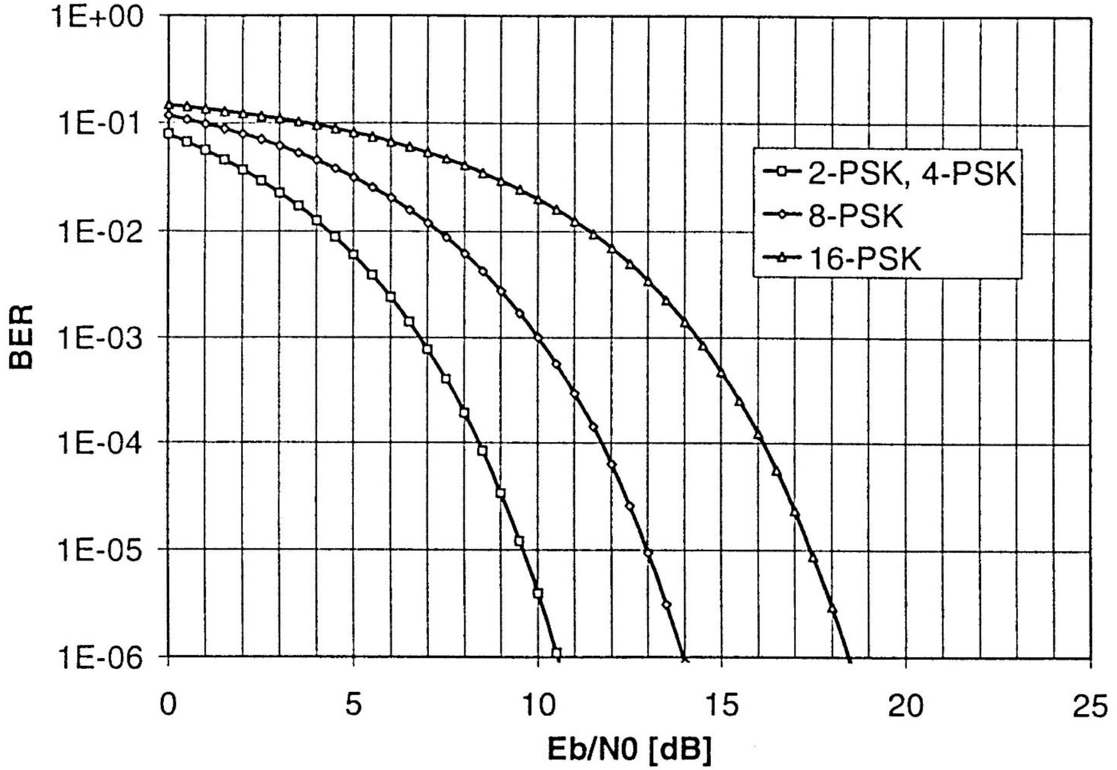
Transmission of 1 symbol over AWGN channel (XII)

BER for M-PAM (matched filter reception)



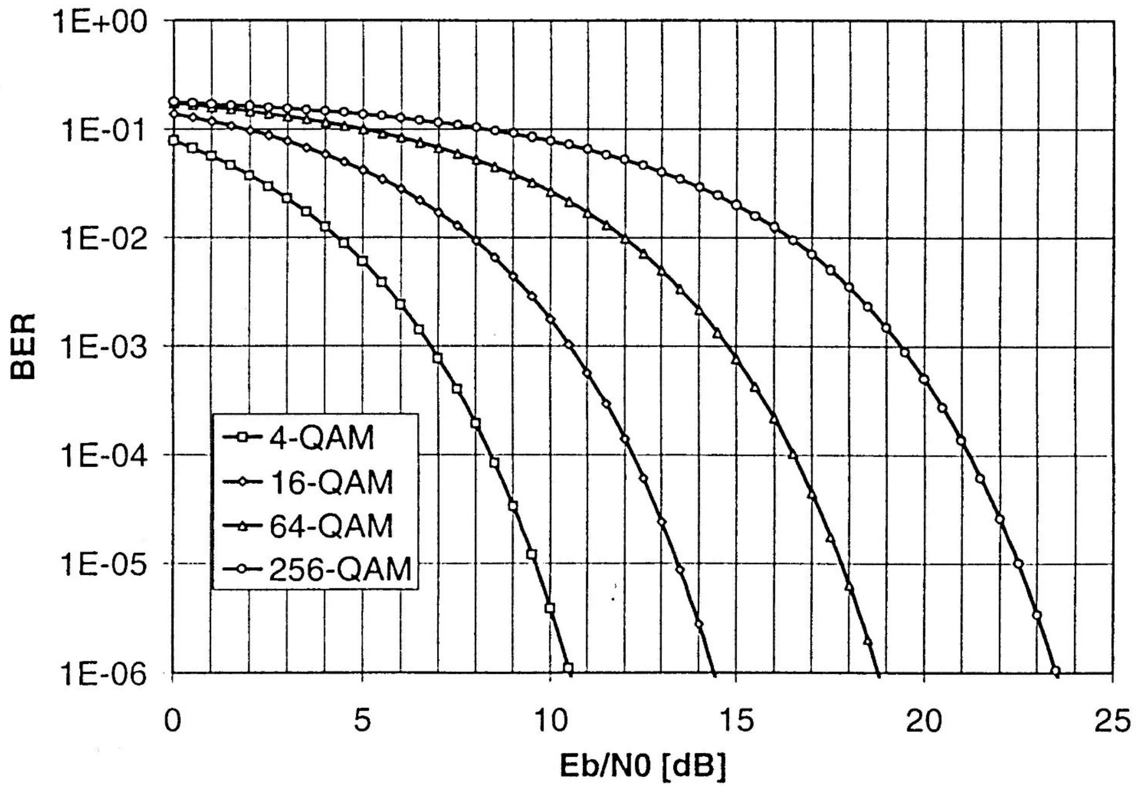
Transmission of 1 symbol over AWGN channel (XIII)

BER for M-PSK (matched filter reception)



Transmission of 1 symbol over AWGN channel (XIV)

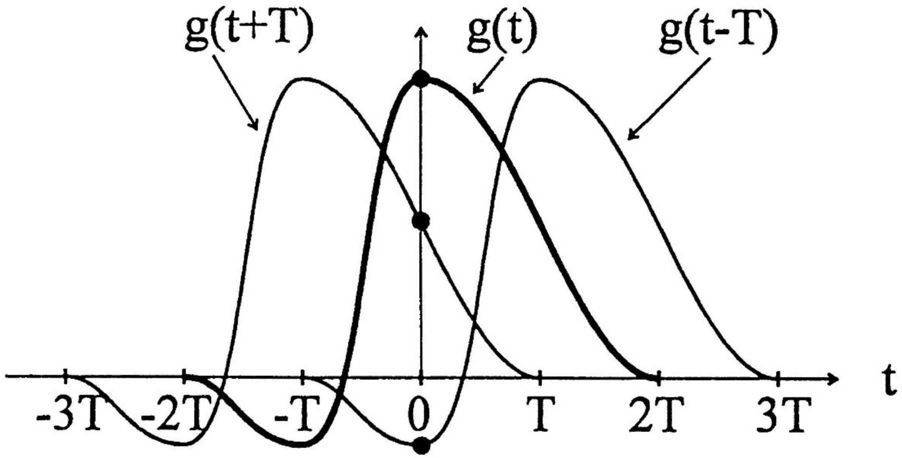
BER for M-QAM (matched filter reception)



Symbol sequence over AWGN channel (I)

- ISI (intersymbol interference) results if

$$\exists m \neq 0 \text{ such that } g(m.T_s) \neq 0$$



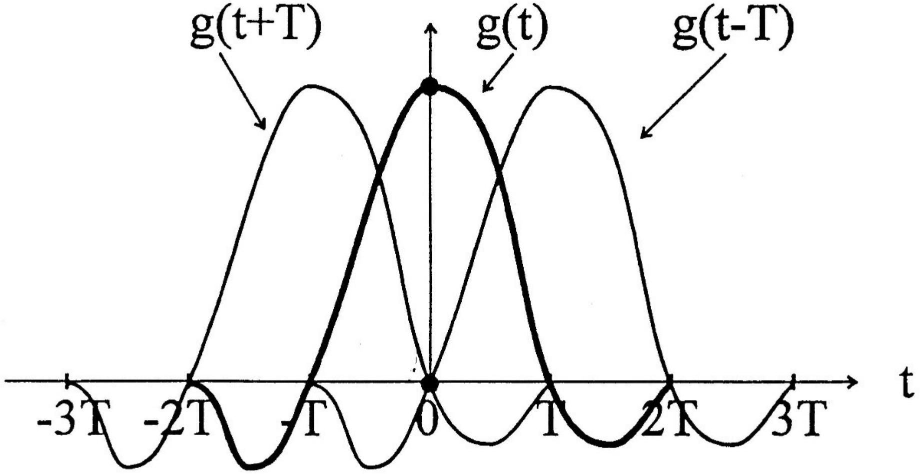
g(t)=p(t)*f(t)

- ISI results in increased BER

Symbol sequence over AWGN channel (II)

- No ISI (intersymbol interference) if

$$\forall m \neq 0 \ : g(m.T_s) = 0$$



- zero ISI -> 1-symbol BER analysis still valid
- design zero-ISI pulses ?

Nyquist ISI Criterion

If we denote the channel impulse response as $h(t)$, then the condition for an ISI-free response can be expressed as:

$$h(nT_s) = \begin{cases} 1; & n = 0 \\ 0; & n \neq 0 \end{cases}$$

for all integers n, where T_s is the symbol period. The Nyquist theorem says that this is equivalent to:

$$\frac{1}{T_s} \sum_{k=-\infty}^{+\infty} H\left(f - \frac{k}{T_s}\right) = 1 \quad \forall f,$$

where $H(f)$ is the Fourier transform of $h(t)$. This is the Nyquist ISI criterion.

This criterion can be intuitively understood in the following way: frequency-shifted replicas of *H(f)* must add up to a constant value.

In practice this criterion is applied to baseband filtering by regarding the symbol sequence as weighted impulses (Dirac delta function). When the baseband filters in the communication system satisfy the Nyquist criterion, symbols can be transmitted over a channel with flat response within a limited frequency band, without ISI. Examples of such baseband filters are the raised-cosine filter, or the sinc filter as the ideal case.

Zero-ISI-forcing pulse design (I)

- **Nyquist Criterion for Zero–ISI**.
  Nyquist proposed a condition for pulses p(t) to have **zero–ISI** when transmitted through a channel with sufficient bandwidth to allow the spectrum of all the transmitted signal to pass.
- No ISI (intersymbol interference) if

$$\forall m \neq 0 \; : g(m.T_s) = 0$$

- Equivalent frequency-domain criterion:

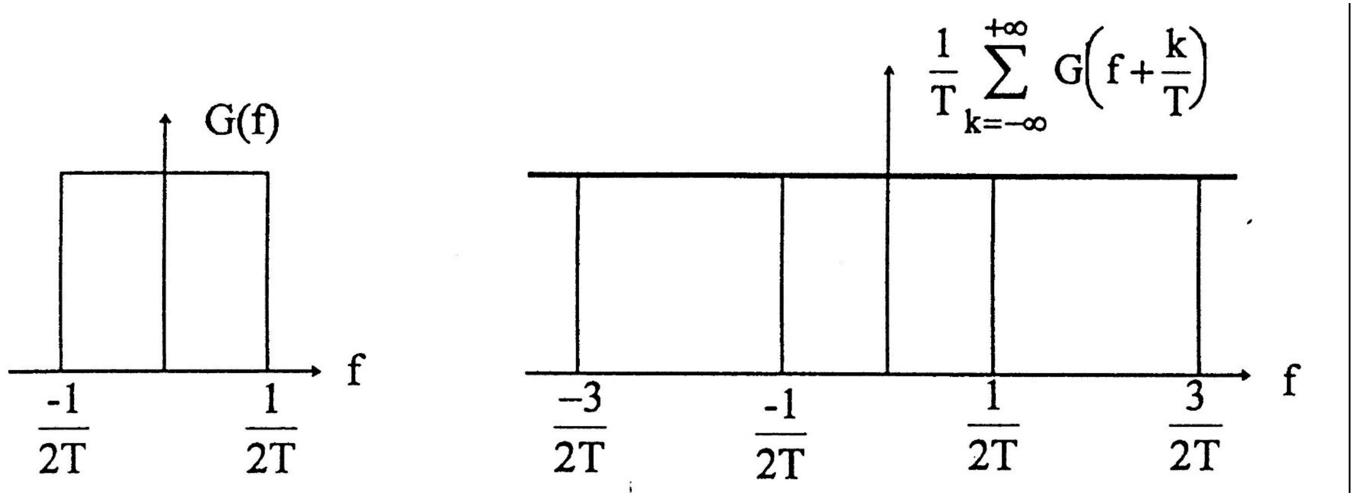
$$\frac{1}{T_s}\sum_{k=-\infty}^{+\infty} G(f + \frac{k}{T_s}) = \text{constant} = g(0)$$

This is called the `Nyquist criterion for zero-ISI’

Pulses that satisfy this criterion are called `Nyquist pulses’

Zero-ISI-forcing pulse design (II)

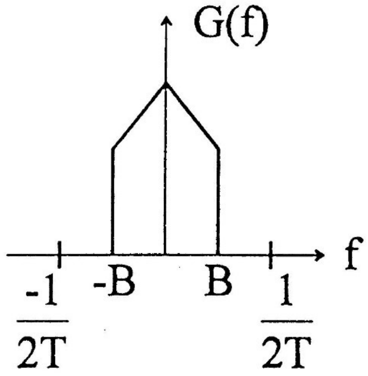
- Nyquist Criterion for Bandwidth = 1/2Ts

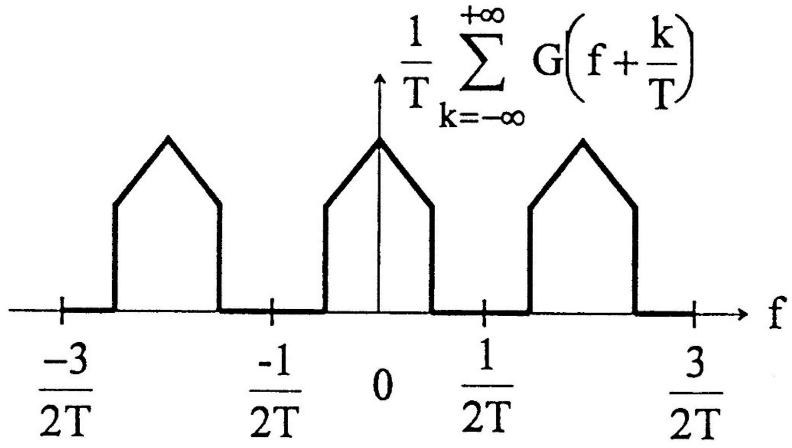


Nyquist criterion can be fulfilled only when G(f) is constant for |f|<B, hence *ideal lowpass filter.*

Zero-ISI-forcing pulse design (III)

- Nyquist Criterion for Bandwidth < 1/2Ts

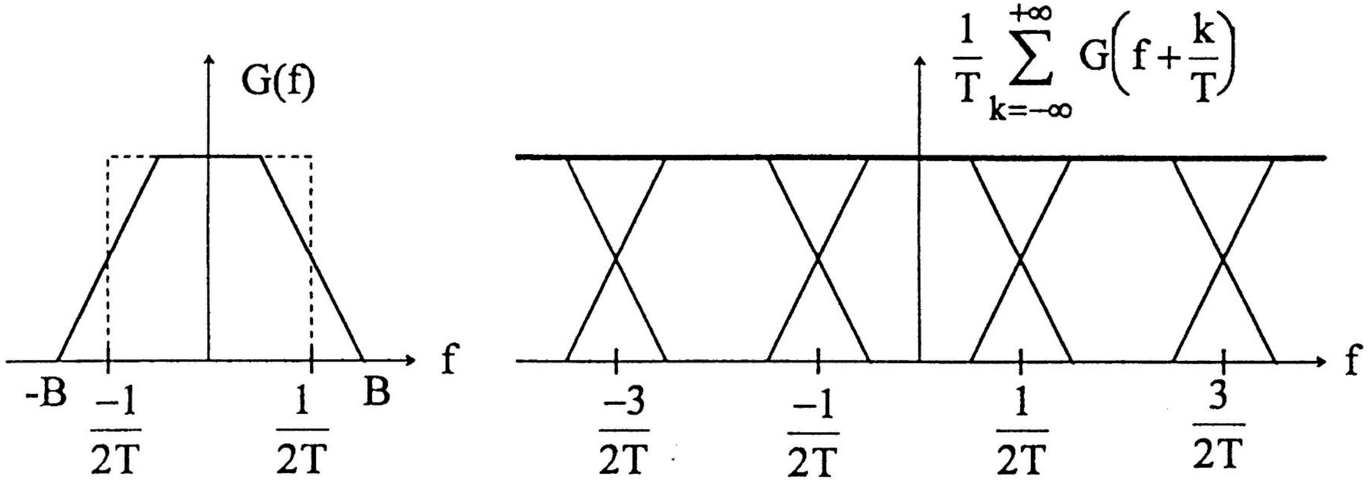




Nyquist criterion can never be fulfilled

Zero-ISI-forcing pulse design (IV)

- Nyquist Criterion for Bandwidth > 1/2Ts



Infinitely many pulses satisfy Nyquist criterion

Zero-ISI-forcing pulse design (V)

- Nyquist Criterion for Bandwidth > 1/2Ts

  practical choices have 1/T>Bandwidth>1/2Ts

Example:

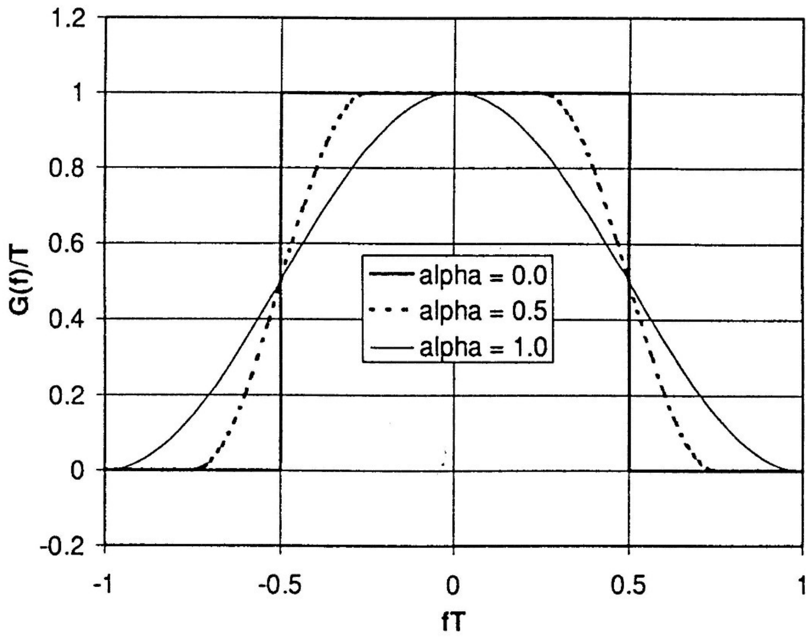
Raised Cosine (RC) Pulses

$$\text{Bandwidth} = \frac{1+\alpha}{2T_s}$$

$$\text{Excess Bandwidth} = \alpha .100\,(\%)$$

$$\alpha : \text{`roll-off factor'}$$

$$0 \leq \alpha \leq 1$$

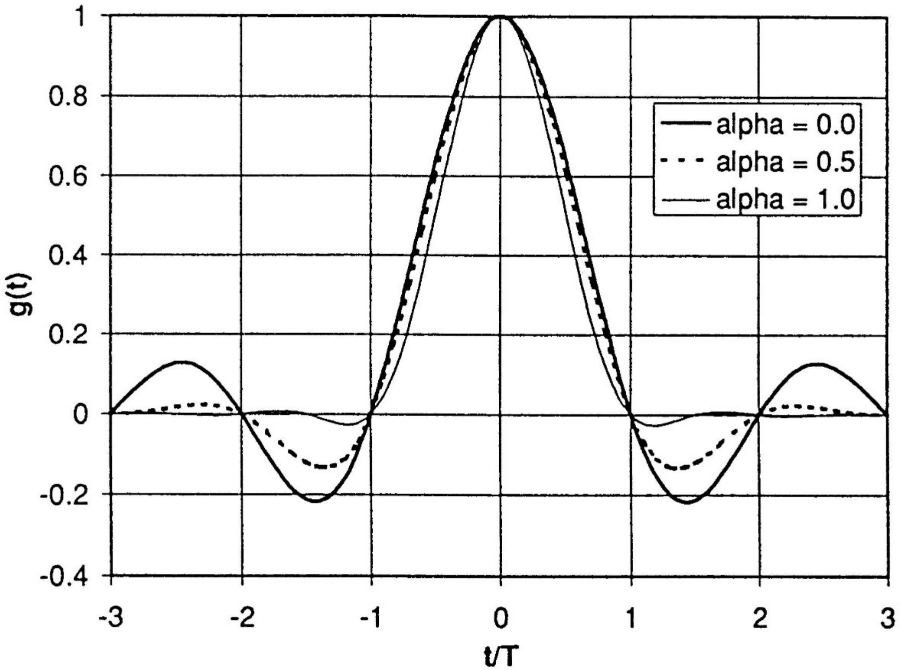


Zero-ISI-forcing pulse design (VI)

Example:

Raised Cosine Pulses

(time-domain)



The roll-off factor, alpha, is a measure of the **excess bandwidth** of the filter, i.e. the **bandwidth** occupied beyond the Nyquist **bandwidth** of . ... This shows that the **excess bandwidth** of the filter can be reduced, but only at the expense of an elongated impulse response.

Zero-ISI-forcing pulse design (VII)

Procedure:

1. Construct Nyquist pulse G(f) (*)

   e.g. G(f) = raised cosine pulse

   (formulas, see Lee & Messerschmitt p.190)

2. Construct F(f) and P(f), such that (**)

   F(f)=P*(f) and P(f).F(f)=G(f) -> P(f).P*(f)=G(f)

   e.g. square-root raised cosine (RRC) pulse

   (formulas, see Lee & Messerschmitt p.228)

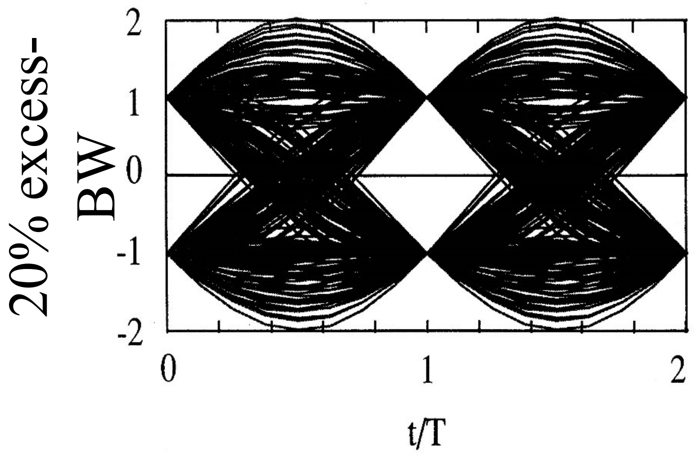
(*) zero-ISI, hence 1-symbol BER performance

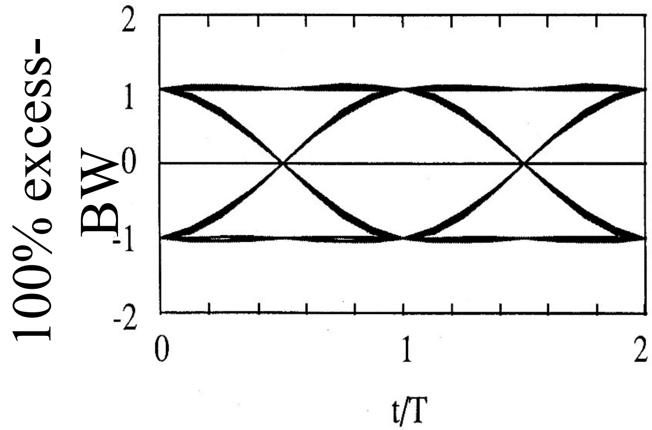
(**) matched filter reception = optimal performance

Zero-ISI-forcing pulse design (VIII)

- Observation: Excess BW simplifies implementation

  - sampling instant less critical (see eye diagrams): An **eye diagram** is a common indicator of the „quality of signals“ in high-speed digital transmissions





`eye diagram’ is `oscilloscope view’ of signal before sampler, when symbol timing serves as a trigger

https://www.edn.com/design/test-and-measurement/4389368/Eye-Diagram-Basics-Reading-and-applying-eye-diagrams

Zero-ISI-forcing pulse design (IX)

- Note: From the eye diagrams, it is seen that selecting a proper sampling instant is crucial

  (for having zero-ISI)

  ->requires accurate clock synchronization,

  a.k.a. `timing recovery', at the receiver

  (clock rate & phase)

  ->`timing recovery' not addressed here

  see e.g. Lee & Messerschmitt, Chapter 17